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A Chaotic Jerk System with Three Cubic Nonlinearities, Dynamical Analysis, Adaptive Chaos Synchronization and Circuit Simulation

S Vaidyanathan1,, A Sambas2 , S Zhang3 , Mujiarto2 , M Mamat4 and Subiyanto5*

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Abstract

A 3-D new chaotic jerk system with three cubic nonlinearities is proposed in this paper. The dynamical properties of the new jerk system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, symmetry, dissipativity, etc. Also, a detailed dynamical analysis of the jerk system has been carried out with bifurcation diagram and Lyapunov exponents. As an engineering application, adaptive synchronization of the new chaotic jerk system with itself is designed via backstepping control method. Furthermore, an electronic circuit realization of the new chaotic jerk system is presented in detail to confirm the feasibility of the theoretical chaotic jerk model. **(Approx. 101 words)**

Keywords: Chaotic Jerk System, three Cubic, Nonlinearities

Topic: Engineering and Technology

Type: Oral Presentation

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Email: iccomset@umtas.ac.id

Date: 11 October 2018

Letter of Acceptance for Abstract

Dear Authors: S Vaidyanathan1,*, A Sambas2 , S Zhang3 , Mujiarto2 , M Mamat4 and Subiyanto5

We are pleased to inform you that your abstract (ABS-227, Oral Presentation), entitled:

"A Chaotic Jerk System with Three Cubic Nonlinearities, Dynamical Analysis, Adaptive Chaos Synchronization and Circuit Simulation"

has been reviewed and accepted to be presented at ICComSET 2018 conference to be held on 27-28 November 2018 in Tasikmalaya, Indonesia.

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Date: 20 October 2018

Letter of Acceptance for Full Paper

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Date: 25 October 2018

Letter of Invitation

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S Vaidyanathan[1,*], A Sambas[2], S Zhang[3], Mujiarto[2], M Mamat[4] and Subiyanto[5]

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[*]sundarvtu@gmail.com

Abstract. A 3-D new chaotic jerk system with three cubic nonlinearities is proposed in this paper. The dynamical properties of the new jerk system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, symmetry, dissipativity, etc. Also, a detailed dynamical analysis of the jerk system has been carried out with bifurcation diagram and Lyapunov exponents. As an engineering application, adaptive synchronization of the new chaotic jerk system with itself is designed via backstepping control method. Furthermore, an electronic circuit realization of the new chaotic jerk system is presented in detail to confirm the feasibility of the theoretical chaotic jerk model.

1. Introduction

Chaos theory deals with chaotic systems which are nonlinear dynamical systems exhibiting high sensitivity to small changes in initial conditions [1-2]. Chaotic systems are very useful in many applications in science and engineering such as temperature model [3], biology [4], physics [5], cellular neural networks [6], satellite [7], robotics [8], encryption [9], finance systems [10], circuits [11-12], etc.

In physics, a jerk ODE can be written as the third order dynamics

$$\frac{d^3x}{dt^3} = \varphi\left(x, \frac{dx}{dt}, \frac{d^2x}{dt^2}\right) \tag{1}$$

In (1), $x(t)$ stands for the displacement, $\frac{dx}{dt}$ the velocity, $\frac{d^2x}{dt^2}$ the acceleration and $\frac{d^3x}{dt^3}$ the jerk.

Thus, we call the ODE (1) as the jerk differential equation.

For qualitative analysis, it is convenient to express the third-order ODE (1) in a system form. This is achieved by defining the following phase variables:

$$\begin{cases} x_1(t) = x(t) \\ x_2(t) = \dot{x}(t) \\ x_3(t) = \ddot{x}(t) \end{cases} \tag{2}$$

Thus, the phase variables in (2) can be viewed as the displacement, velocity and acceleration, respectively. Using them, we can express the jerk differential equation (1) as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = \varphi(x_1, x_2, x_3) \end{cases} \tag{3}$$

Many jerk systems have been found in the chaos literature [13-17]. Jerk systems have important applications in mechanical engineering [1-2]. Some famous jerk systems are Sprott systems [13], Li system [14], Elsonbaty system [15], Coullet system [16], Vaidyanathan systems [17], etc.

In this research paper, we report the finding of a new chaotic jerk system with three cubic nonlinearities. We describe the phase plots of the jerk system and do a rigorous dynamic analysis by finding bifurcation diagrams, Lyapunov exponents, Kaplan-Yorke dimension, symmetry analysis, etc.

As a control application, we derive new results for the adaptive synchronization of the new chaotic jerk system with itself with unknown parameters. Synchronization of chaotic systems deals with the control problem of finding suitable feedback control laws so as to asymptotically synchronize the respective trajectories of a pair of chaotic systems called as master and slave systems. We use backstepping control method for achieving global chaos synchronization of the new chaotic jerk system with itself. Backstepping control method is a recursive procedure used for stabilizing nonlinear dynamical systems.

Section 2 describes the new chaotic jerk system, its phase plots and Lyapunov exponents. Section 3 describes the dynamic analysis of the new chaotic jerk system. Section 4 describes the backstepping-based adaptive synchronization of the new chaotic jerk system with itself. Furthermore, an electronic circuit realization of the new chaotic system is presented in detail in Section 5. The circuit experimental results of the new chaotic jerk system in Section 5 agreement with the numerical simulations via MATLAB obtained in Section 2. Section 6 contains the main conclusions.

2. A new chaotic jerk system with three cubic nonlinearities

In this work, we report a new 3-D chaotic jerk system given by the dynamics

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -x_1 - ax_3 - bx_1^2 x_3 + x_1 x_2^2 - x_1^3 \end{cases} \tag{4}$$

where x_1, x_2, x_3 are state variables and a, b are positive constants.

In this paper, we show that the jerk system (1) is *chaotic* for the parameter values

$$a = 2, \quad b = 0.2 \tag{5}$$

For numerical simulations, we take the initial values of the jerk system (4) as $X(0) = (0.2, 0.2, 0.2)$. Figure 1 shows the phase portraits of the strange attractor of the new chaotic jerk system (4) for $(a,b) = (2, 0.2)$ and initial conditions $X(0) = (0.2, 0.2, 0.2)$. Figure 1 (a) shows the 3-D phase portrait of the new chaotic jerk system (4). Figures 1 (b)-(c) show the projections of the new chaotic system (4) in (x_1, x_2), (x_2, x_3) and (x_1, x_3) coordinate planes, respectively.

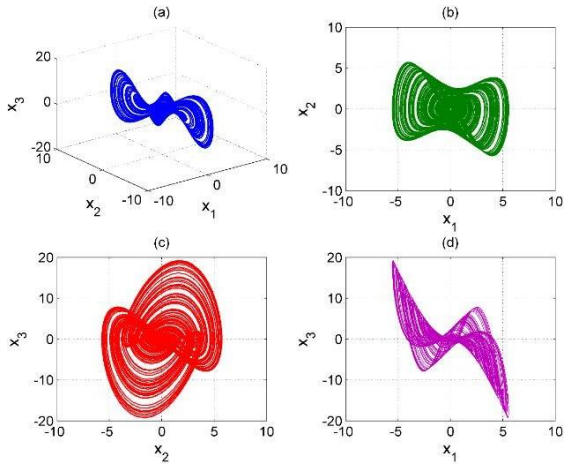


Figure 1. Plots of the chaotic jerk system (4) for $(a,b)=(2,0.2)$ and $X(0)=(0.2,0.2,0.2)$

For the rest of this section, we take the values of the parameters a and b as in the chaotic case (5), *i.e.* $(a,b)=(2,0.2)$

The equilibrium points of the new chaotic jerk system (4) are obtained by solving the system of equations

$$x_2 = 0 \tag{6a}$$

$$x_3 = 0 \tag{6b}$$

$$-x_1 - ax_3 - bx_1^2 x_3 + x_1 x_2^2 - x_1^3 = 0 \tag{6c}$$

From (6a) and (6b), we deduce that $x_2 = x_3 = 0.$

Substituting these in (6c), we obtain $-x_1\left(1+x_1^2\right)=0.$ This gives $x_1=0.$

Hence, $E_0=(0,0,0)$ is the unique equilibrium of the chaotic jerk system (4).

The Jacobian matrix of the new jerk system (4) at $E_0=(0,0,0)$ is obtained as

$$J=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -2 \end{bmatrix} \tag{7}$$

The Jacobian matrix J has the spectral values $-2.2056,\ 0.1028\pm 0.6655i.$

This shows that the equilibrium point E_0 is a saddle-focus and unstable.

We note that the new chaotic system (4) is invariant under the coordinates transformation

$$\left(x_1,x_2,x_3\right)\mapsto\left(-x_1,-x_2,-x_3\right) \tag{8}$$

for all values of the parameters. This shows that the new chaotic system (4) has point-reflection symmetry about the equilibrium $E_0=(0,0,0).$

For the parameter values as in the chaotic case (5) and the initial state $X(0)=(0.2,0.2,0.2),$ the Lyapunov exponents of the new jerk system (4) are determined using Wolf's algorithm as

$$LE_1=0.1680,\ LE_2=0,\ LE_3=-2.6705 \tag{9}$$

The jerk system (4) is chaotic since $LE_1=0.1680>0.$ Thus, the system (4) exhibits a self-excited strange chaotic attractor. Also, we note that the sum of the Lyapunov exponents in (9) is negative. This shows that the jerk system (4) is dissipative.

The Kaplan-Yorke dimension of the jerk system (4) is determined as

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.0629, \tag{10}$$

which indicates the high complexity of the chaotic jerk system (4).

Figure 2 shows the Lyapunov exponents of the new chaotic jerk system (4) with self-excited, strange chaotic attractor.

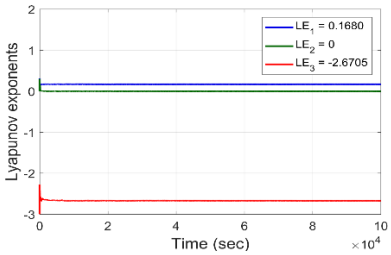
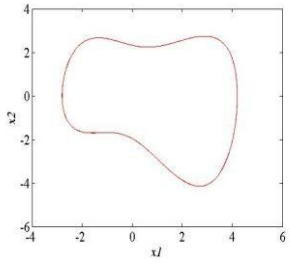


Figure 2. Lyapunov exponents of the new chaotic jerk system (4) for $(a,b) = (2, 0.2)$

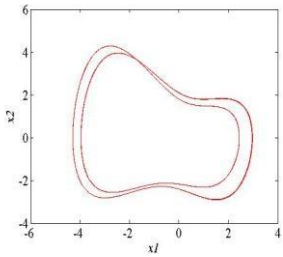
3. Bifurcation Analysis for the New Chaotic Jerk System

In this section, we describe a bifurcation analysis for the new chaotic jerk system (4) introduced in Section 2. Bifurcation analysis is an important topic for understanding chaotic systems [18-22].

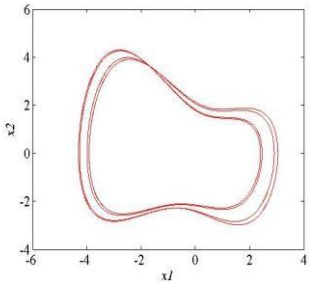
Some sample results are plotted to verify the reversed period-doubling route to chaos in Figure 3.



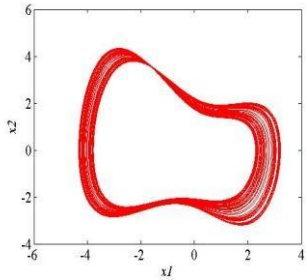
(a) $a = 3$, period-1



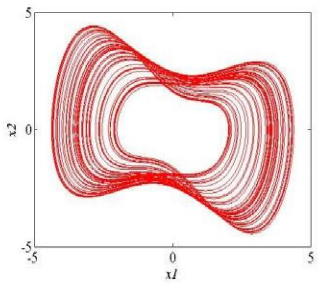
(b) $a = 2.86$, period-2



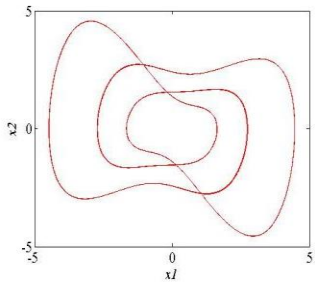
(c) $a = 2.85$, period-4



(d) $a = 2.8$, chaos



(e) $a = 2.7$, chaos



(f) $a = 2.58$, period-3

Figure 3. Some sample plots of the jerk system (4) for different values of a when we fix $b = 0.2$

Next, we fix $b = 0.2$, the initial state as $X(0) = (0.2, 0.2, 0.2)$ and vary a in $[2,3]$.

From the bifurcation diagram given in Figure 4, we see that new chaotic jerk system (4) exhibits chaos, period, reversed period-doubling route, as well as several periodic windows. Note that there is a large period-3 periodic window.

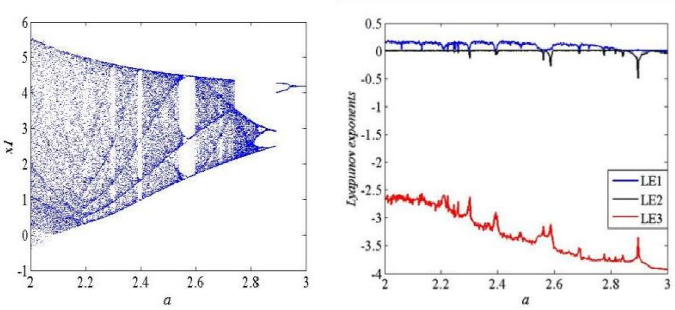


Figure 4. Bifurcation diagram and Lyapunov exponents for the new chaotic jerk system (4) when we fix $b = 0.2$ and vary a in the interval $[2,3]$

Next, we fix $a = 2$, initial state as $X(0) = (0.2, 0.2, 0.2)$ and vary b in $[0.2, 0.45]$. The corresponding bifurcation diagram and Lyapunov exponents for the new chaotic jerk system (4) are shown in Figure 5. Obviously, from the bifurcation diagram, one can get that the system shows a reversed period-doubling route to chaos in the whole region. Note that there is a large period-3 periodic window.

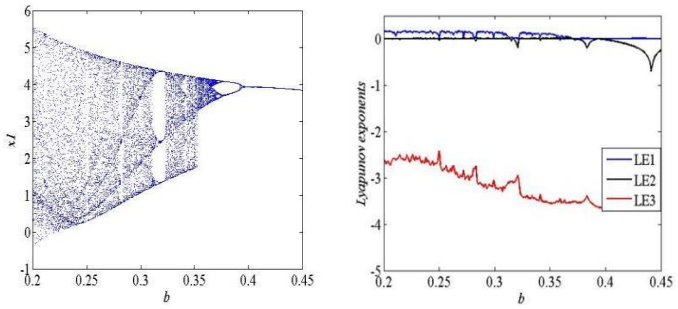


Figure 5. Bifurcation diagram and Lyapunov exponents for the new chaotic jerk system (4) when we fix $a = 2$ and vary b in the interval $[0.2, 0.45]$

Next, we describe our results for coexisting attractors. We note that the blue color denotes the trajectory of the new chaotic jerk system (4) starting from $X_0 = (0.2, 0.2, 0.2)$ and the red color denotes the trajectory of the new chaotic jerk system (4) starting from $Y_0 = (-0.2, -0.2, -0.2)$.

We fix $b = 0.2$ and vary a in the interval $[2,3]$. As can be seen from the bifurcation diagram given in Figure 6, there exists coexisting attractors in the region of $[2.75, 2.9]$.

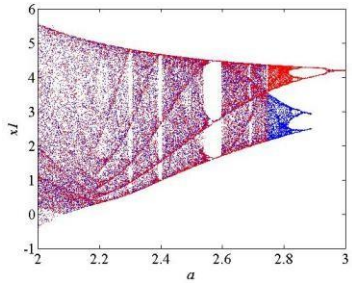
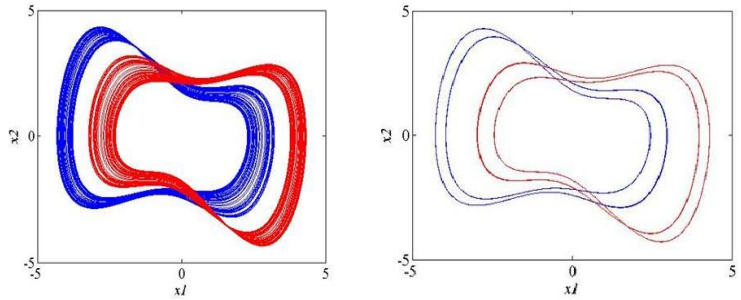


Figure 6. Bifurcation diagram for the new chaotic jerk system (4) when we fix $b = 0.2$ and vary a in the interval $[2,3]$.



(a) $a = 2.8, b = 0.2$ (b) $a = 2.86, b = 0.2$

Figure 7. (a) When $a = 2.8$, coexisting chaotic attractors, and (b) when $a = 2.86$, coexisting period-2 attractors for the new chaotic jerk system (4). We fix $b = 0.2$.

4. Backstepping-Based Global Chaos Synchronization of the Chaotic Jerk Systems

In this section, we use backstepping control method to achieve global chaos synchronization of the new chaotic jerk systems.

As the master system for the synchronization, we consider the new chaotic jerk system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -x_1 - ax_3 - bx_1^2 x_3 + x_1 x_2^2 - x_1^3 \end{cases} \tag{11}$$

In (11), x_1, x_2, x_3 are the states and a, b are unknown state parameters.

As the slave system for the synchronization, we consider the new chaotic jerk system

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = -y_1 - ay_3 - by_1^2 y_3 + y_1 y_2^2 - y_1^3 + u \end{cases} \tag{12}$$

In (12), y_1, y_2, y_3 are the states and u is a backstepping control to be designed.

The synchronization error between the jerk systems (11) and (12) can be classified as follows:

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \quad e_3 = y_3 - x_3 \tag{13}$$

We find the error dynamics as follows:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = -e_1 - ae_3 - b(y_1^2 y_3 - x_1^2 x_3) + y_1 y_2^2 - x_1 x_2^2 - y_1^3 + x_1^3 + u \end{cases} \tag{14}$$

We denote $A(t), B(t)$ as estimates for the unknown parameters a, b, respectively.

The error between the parameters and their estimates is defined as follows:

$$e_a(t) = a - A(t), \quad e_b(t) = b - B(t) \tag{15}$$

It is easy to see that

$$\begin{cases} \dot{e}_a = -\dot{A}(t) \\ \dot{e}_b = -\dot{B}(t) \end{cases} \tag{16}$$

Using adaptive backstepping control, we establish a key result of this section.

Theorem 1. The master and slave chaotic jerk systems represented by (11) and (12) with unknown parameters a and b are globally and exponentially synchronized by means of the adaptive backstepping controller using estimates $A(t)$ and $B(t)$ given by

$$u = -2e_1 - 5e_2 - [3 - A(t)]e_3 + B(t)(y_1^2 y_3 - x_1^2 x_3) - y_1 y_2^2 + x_1 x_2^2 + y_1^3 - x_1^3 - K\sigma_3 \tag{17}$$

where $K > 0$ is a gain constant,

$$\sigma_3 = 2e_1 + 2e_2 + e_3 \tag{18}$$

and the update law for the estimates $A(t), B(t)$ is given by

$$\begin{cases} \dot{A} = -\sigma_3 e_3 \\ \dot{B} = -\sigma_3 \left(y_1^2 y_3 - x_1^2 x_3 \right) \end{cases} \tag{19}$$

Proof. The result is proved via backstepping control method, which is a recursive procedure in Lyapunov stability theory. We start with the Lyapunov function

$$V_1(\sigma_1) = 0.5\ \sigma_1^2 \tag{20}$$

where $\sigma_1 = e_1$.

Differentiating V_1 along the error dynamics (14), we get

$$\dot{V}_1 = \sigma_1 \dot{\sigma}_1 = e_1 \dot{e}_1 = -e_1^2 + e_1 \left(e_1 + e_2 \right) \tag{21}$$

We define

$$\sigma_2 = e_1 + e_2 \tag{22}$$

Using Eq. (22), we can simplify Eq. (21) as

$$\dot{V}_1 = -\sigma_1^2 + \sigma_1 \sigma_2 \tag{23}$$

Next, we define the Lyapunov function

$$V_2 \left(\sigma_1, \sigma_2 \right) = V_1 (\sigma_1) + 0.5 \sigma_2^2 = 0.5 \left(\sigma_1^2 + \sigma_2^2 \right) \tag{24}$$

Differentiating V_2 along the error dynamics (14), we get

$$\dot{V}_2 = -\sigma_1^2 - \sigma_2^2 + \sigma_2 \left(2e_1 + 2e_2 + e_3 \right) \tag{25}$$

We define

$$\sigma_3 = 2e_1 + 2e_2 + e_3 \tag{26}$$

Using (26), we can express (25) as

$$\dot{V}_2 = -\sigma_1^2 - \sigma_2^2 + \sigma_2 \sigma_3 \tag{27}$$

To simplify the notation, we set $\sigma = (\sigma_1, \sigma_2, \sigma_3)$.

Finally, we define the quadratic Lyapunov function

$$V(\sigma, e_a, e_b) = 0.5 \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 \right) + 0.5 \left(e_a^2 + e_b^2 \right) \tag{28}$$

It is evident that V is a positive definite function on R^5.

Differentiating V along the error dynamics (14) and (16), we get

$$\dot{V} = -\sigma_1^2 - \sigma_2^2 - \sigma_3^2 + \sigma_3 T - e_a \dot{A} - e_b \dot{B} \tag{29}$$

where

$$T = \dot{\sigma}_3 + \sigma_2 + \dot{\sigma}_3 = \dot{\sigma}_3 + \sigma_2 + \left(2\dot{e}_1 + 2\dot{e}_2 + e_3 \right) \tag{30}$$

A simple calculation shows that

$$T = 2e_1 + 5e_2 + (3-a)e_3 - b \left(y_1^2 y_3 - x_1^2 x_3 \right) + y_1 y_2^2 - x_1 x_2^2 - y_1^3 + x_1^3 + u \tag{31}$$

Substituting the value of u from Eq. (17) into Eq. (31), we obtain

$$T = -[a - A(t)]e_3 - [b - B(t)] \left(y_1^2 y_3 - x_1^2 x_3 \right) - K\sigma_3 \tag{32}$$

We can simplify Eq. (32) by using the definitions given in Eq. (15) as follows:

$$T = -e_a e_3 - e_b \left(y_1^2 y_3 - x_1^2 x_3 \right) - K\sigma_3 \tag{33}$$

Substituting the value of T from Eq. (33) into Eq. (29), we get

$$\dot{V} = -\sigma_1^2 - \sigma_2^2 - (1+K)\sigma_3^2 + e_a(-\sigma_3 e_3 - \dot{A}) + e_b \left[-\sigma_3 \left(y_1^2 y_3 - x_1^2 x_3 \right) - \dot{B} \right] \tag{34}$$

Implementing the parameter update law (19) into Eq. (34), we get

$$\dot{V} = -\sigma_1^2 - \sigma_2^2 - (1+K)\sigma_3^2 \tag{35}$$

which is a negative semi-definite function on R^5.

Thus, by Barbalat's lemma [23] in Lyapunov stability theory, we conclude that $\sigma(t) \to 0$ exponentially as $t \to \infty$. Hence, it follows that $e(t) \to 0$ as $t \to \infty$.

This completes the proof.

For numerical plots, we take the constants (a,b) as in the chaotic case, *viz.* $(a,b) = (2, 0.2)$.

We take the gain as $K = 10$.

The initial conditions of the master jerk system (11) are picked as

$$x_1(0) = 2.6, \quad x_2(0) = 0.3, \quad x_3(0) = 1.2 \tag{36}$$

The initial conditions of the slave jerk system (12) are taken as

$$y_1(0) = 1.7, \quad y_2(0) = 3.8, \quad y_3(0) = 4.9 \tag{37}$$

The initial conditions of the parameter estimates are taken as

$$A(0) = 2.8, \; B(0) = 7.4 \tag{38}$$

Figure 8 shows the global chaos synchronization of the chaotic jerk systems (11) and (12).

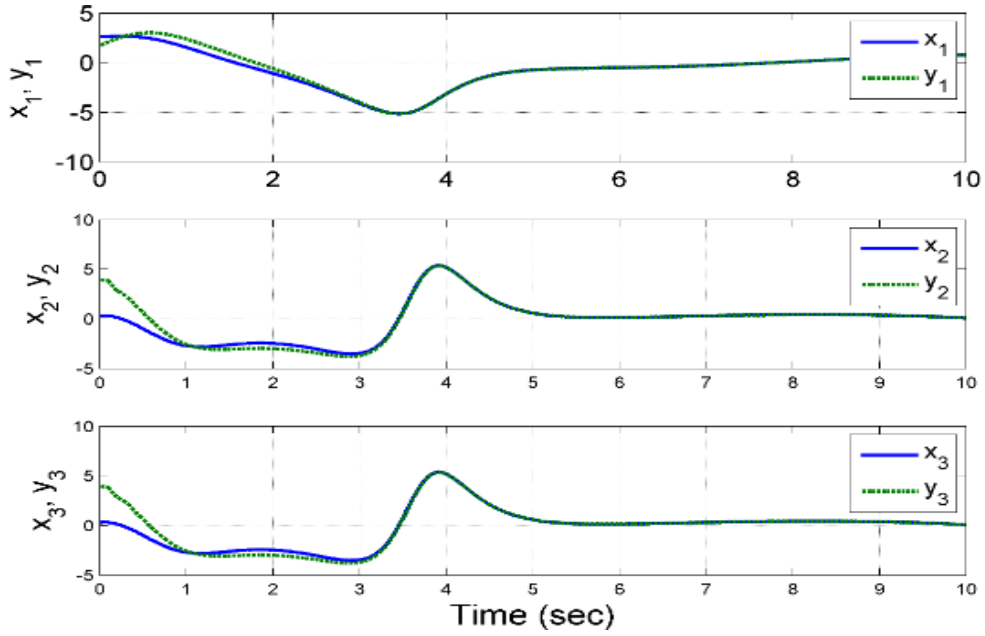


Figure 8 Synchronization of the chaotic jerk systems (11) and (12)

5. Circuit Implementation of the New Chaotic Jerk System

In this work, we describe a realization of theoretical jerk model (4) by using electronic components. As shown in Figure 9, the circuit includes three op-amp integrator circuits based on three operational amplifiers (U1A, U2A, U3A), three inverting amplifiers (U5A, U6A, U7A) which are implemented with the operational amplifier TL082CD and five multipliers by using AD633JN.

The circuital equations of the designed jerk circuit are given by

$$\begin{cases} \dot{x}_1 = \dfrac{1}{C_1 R_1} x_2 \\ \dot{x}_2 = \dfrac{1}{C_2 R_2} x_3 \\ \dot{x}_3 = -\dfrac{1}{C_3 R_3} x_1 - \dfrac{1}{C_3 R_4} x_3 - \dfrac{1}{100 C_3 R_5} x_1^2 x_3 + \dfrac{1}{100 C_3 R_6} x_1 x_2^2 - \dfrac{1}{100 C_3 R_7} x_1^3 \end{cases} \tag{39}$$

where x , y , and z are the voltages across the capacitors C_1, C_2 and C_3, respectively. Equations (39) match Eqs. (1) when the circuit components are selected as follows: R_4 = 50 kfi, R_5 = 5 kfi, $R_6 = R_7$ = 1 kfi, $R_1 = R_2 = R_3 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13}$ = 100 kfi, $C_1 = C_2 = C_3$ = 1 nF. MultiSIM phase portraits of the circuit are represented in Figure 10. Once more a very good qualitative agreement can be observed between numerical simulations (see Figure 1) and MultiSIM results (see Figure 10).

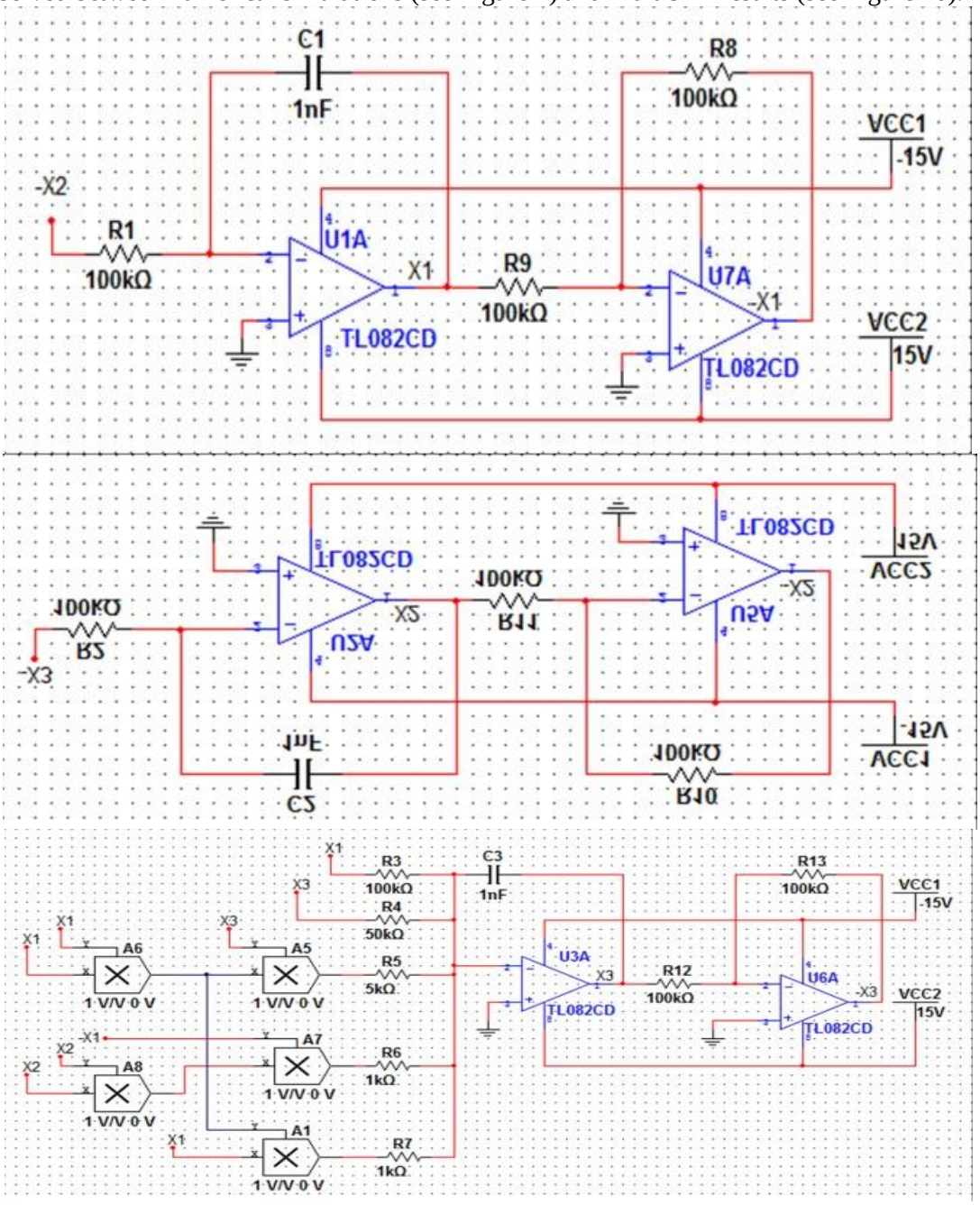
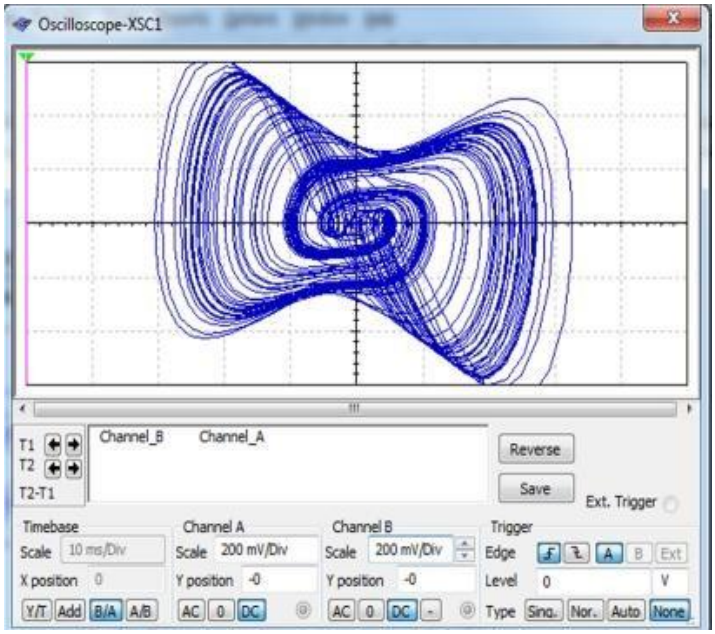
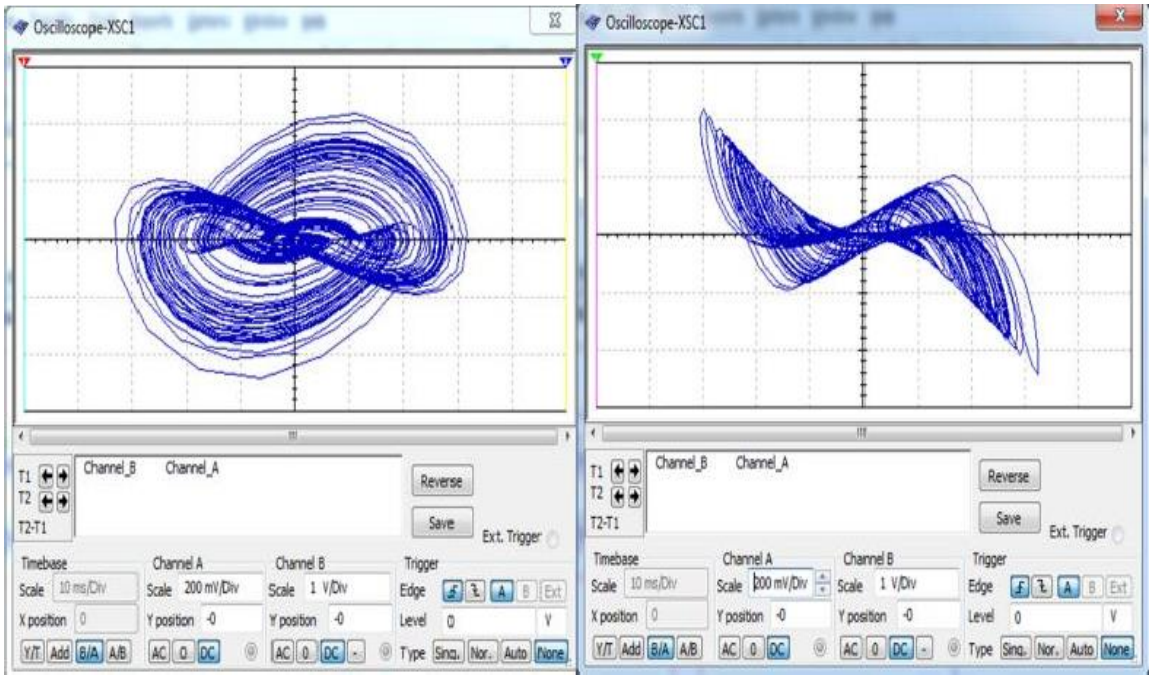


Figure 9. The electronic circuit schematic of new chaotic jerk system



(a)



(b) (c)

Figure 10. MultiSIM chaotic attractors of the new chaotic jerk system (4) (a) x_1- x_2 plane (b) x_2- x_3 plane and (c) x_1-x_3 plane.

6. Conclusions

A new chaotic jerk system with three cubic nonlinearities is proposed in this paper. The dynamical properties of the new jerk system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, symmetry, dissipativity, etc. Also, a detailed dynamical analysis of the jerk system was done using bifurcation diagram and Lyapunov exponents, and we exhibited coexisting chaotic attractors for the new jerk system. The adaptive backstepping-based synchronization of the

new chaotic jerk system with itself was designed. Furthermore, an electronic circuit realization of the new jerk system was shown to confirm the feasibility of the jerk system.

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A Chaotic Jerk System with Three Cubic Nonlinearities, Dynamical Analysis, Adaptive Chaos Synchronization and Circuit Simulation

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Abstract. A 3-D new chaotic jerk system with three cubic nonlinearities is proposed in this paper. The dynamical properties of the new jerk system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, symmetry, dissipativity, etc. Also, a detailed dynamical analysis of the jerk system has been carried out with bifurcation diagram and Lyapunov exponents. As an engineering application, adaptive synchronization of the new chaotic jerk system with itself is designed via backstepping control method. Furthermore, an electronic circuit realization of the new chaotic jerk system is presented in detail to confirm the feasibility of the theoretical chaotic jerk model.

1. Introduction

Chaos theory deals with chaotic systems which are nonlinear dynamical systems exhibiting high sensitivity to small changes in initial conditions [1-2]. Chaotic systems are very useful in many applications in science and engineering such as temperature model [3], biology [4], physics [5], cellular neural networks [6], satellite [7], robotics [8], encryption [9], finance systems [10], circuits [11-12], etc.

In physics, a jerk ODE can be written as the third order dynamics

$$\frac{d^3x}{dt^3} = \varphi\left(x, \frac{dx}{dt}, \frac{d^2x}{dt^2}\right) \tag{1}$$

In (1), $x(t)$ stands for the displacement, $\frac{dx}{dt}$ the velocity, $\frac{d^2x}{dt^2}$ the acceleration and $\frac{d^3x}{dt^3}$ the jerk.

Thus, we call the ODE (1) as the jerk differential equation.

For qualitative analysis, it is convenient to express the third-order ODE (1) in a system form. This is achieved by defining the following phase variables:

$$\begin{cases} x_1(t) = x(t) \\ x_2(t) = \dot{x}(t) \\ x_3(t) = \ddot{x}(t) \end{cases} \tag{2}$$

Thus, the phase variables in (2) can be viewed as the displacement, velocity and acceleration, respectively. Using them, we can express the jerk differential equation (1) as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = \varphi(x_1, x_2, x_3) \end{cases} \tag{3}$$

Many jerk systems have been found in the chaos literature [13-17]. Jerk systems have important applications in mechanical engineering [1-2]. Some famous jerk systems are Sprott systems [13], Li system [14], Elsonbaty system [15], Coullet system [16], Vaidyanathan systems [17], etc.

In this research paper, we report the finding of a new chaotic jerk system with three cubic nonlinearities. We describe the phase plots of the jerk system and do a rigorous dynamic analysis by finding bifurcation diagrams, Lyapunov exponents, Kaplan-Yorke dimension, symmetry analysis, etc.

As a control application, we derive new results for the adaptive synchronization of the new chaotic jerk system with itself with unknown parameters. Synchronization of chaotic systems deals with the control problem of finding suitable feedback control laws so as to asymptotically synchronize the respective trajectories of a pair of chaotic systems called as master and slave systems. We use backstepping control method for achieving global chaos synchronization of the new chaotic jerk system with itself. Backstepping control method is a recursive procedure used for stabilizing nonlinear dynamical systems.

Section 2 describes the new chaotic jerk system, its phase plots and Lyapunov exponents. Section 3 describes the dynamic analysis of the new chaotic jerk system. Section 4 describes the backstepping-based adaptive synchronization of the new chaotic jerk system with itself. Furthermore, an electronic circuit realization of the new chaotic system is presented in detail in Section 5. The circuit experimental results of the new chaotic jerk system in Section 5 agreement with the numerical simulations via MATLAB obtained in Section 2. Section 6 contains the main conclusions.

2. A new chaotic jerk system with three cubic nonlinearities

In this work, we report a new 3-D chaotic jerk system given by the dynamics

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -x_1 - ax_3 - bx_1^2 x_3 + x_1 x_2^2 - x_1^3 \end{cases} \tag{4}$$

where x_1, x_2, x_3 are state variables and a, b are positive constants.

In this paper, we show that the jerk system (1) is *chaotic* for the parameter values

$$a = 2, \quad b = 0.2 \tag{5}$$

For numerical simulations, we take the initial values of the jerk system (4) as $X(0) = (0.2, 0.2, 0.2)$. Figure 1 shows the phase portraits of the strange attractor of the new chaotic jerk system (4) for $(a,b) = (2, 0.2)$ and initial conditions $X(0) = (0.2, 0.2, 0.2)$. Figure 1 (a) shows the 3-D phase portrait of the new chaotic jerk system (4). Figures 1 (b)-(c) show the projections of the new chaotic system (4) in (x_1, x_2), (x_2, x_3) and (x_1, x_3) coordinate planes, respectively.

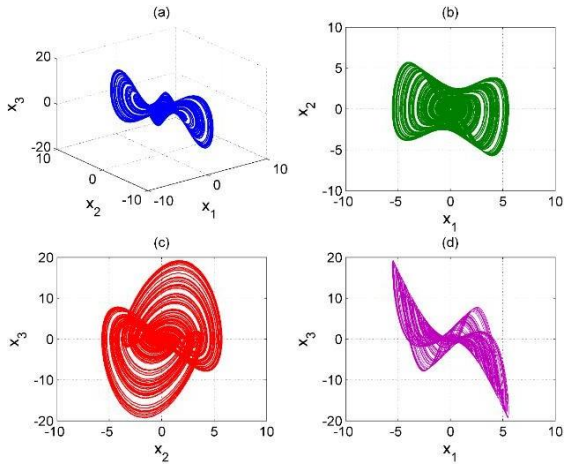


Figure 1. Plots of the chaotic jerk system (4) for $(a,b)=(2,0.2)$ and $X(0)=(0.2,0.2,0.2)$

For the rest of this section, we take the values of the parameters a and b as in the chaotic case (5), *i.e.* $(a,b)=(2,0.2)$

The equilibrium points of the new chaotic jerk system (4) are obtained by solving the system of equations

$$x_2 = 0 \tag{6a}$$

$$x_3 = 0 \tag{6b}$$

$$-x_1 - ax_3 - bx_1^2 x_3 + x_1 x_2^2 - x_1^3 = 0 \tag{6c}$$

From (6a) and (6b), we deduce that $x_2 = x_3 = 0.$

Substituting these in (6c), we obtain $-x_1\left(1+x_1^2\right)=0.$ This gives $x_1=0.$

Hence, $E_0=(0,0,0)$ is the unique equilibrium of the chaotic jerk system (4).

The Jacobian matrix of the new jerk system (4) at $E_0=(0,0,0)$ is obtained as

$$J=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -2 \end{bmatrix} \tag{7}$$

The Jacobian matrix J has the spectral values $-2.2056,\ 0.1028\pm 0.6655i.$

This shows that the equilibrium point E_0 is a saddle-focus and unstable.

We note that the new chaotic system (4) is invariant under the coordinates transformation

$$\left(x_1,x_2,x_3\right)\mapsto\left(-x_1,-x_2,-x_3\right) \tag{8}$$

for all values of the parameters. This shows that the new chaotic system (4) has point-reflection symmetry about the equilibrium $E_0=(0,0,0).$

For the parameter values as in the chaotic case (5) and the initial state $X(0)=(0.2,0.2,0.2),$ the Lyapunov exponents of the new jerk system (4) are determined using Wolf's algorithm as

$$LE_1=0.1680,\ LE_2=0,\ LE_3=-2.6705 \tag{9}$$

The jerk system (4) is chaotic since $LE_1=0.1680>0.$ Thus, the system (4) exhibits a self-excited strange chaotic attractor. Also, we note that the sum of the Lyapunov exponents in (9) is negative. This shows that the jerk system (4) is dissipative.

The Kaplan-Yorke dimension of the jerk system (4) is determined as

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.0629, \tag{10}$$

which indicates the high complexity of the chaotic jerk system (4).

Figure 2 shows the Lyapunov exponents of the new chaotic jerk system (4) with self-excited, strange chaotic attractor.

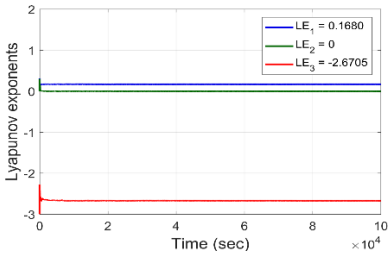
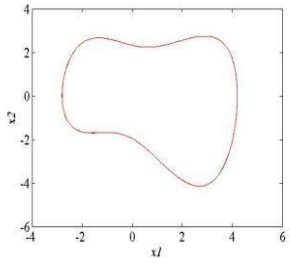


Figure 2. Lyapunov exponents of the new chaotic jerk system (4) for $(a,b) = (2, 0.2)$

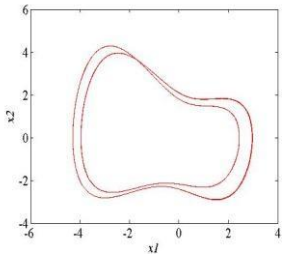
3. Bifurcation Analysis for the New Chaotic Jerk System

In this section, we describe a bifurcation analysis for the new chaotic jerk system (4) introduced in Section 2. Bifurcation analysis is an important topic for understanding chaotic systems [18-22].

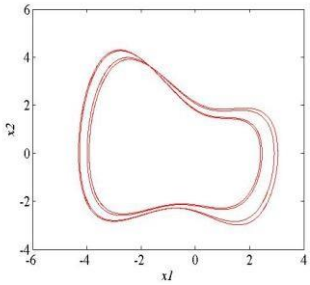
Some sample results are plotted to verify the reversed period-doubling route to chaos in Figure 3.



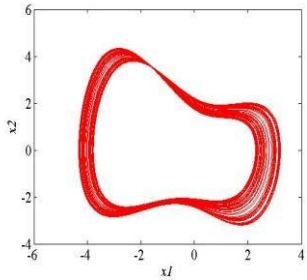
(a) $a = 3$, period-1



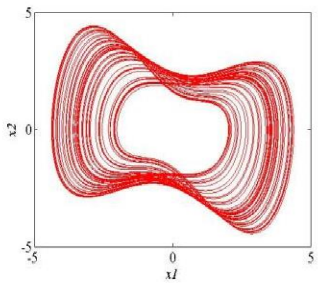
(b) $a = 2.86$, period-2



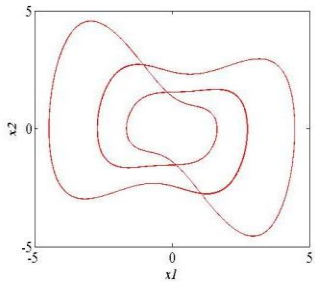
(c) $a = 2.85$, period-4



(d) $a = 2.8$, chaos



(e) $a = 2.7$, chaos



(f) $a = 2.58$, period-3

Figure 3. Some sample plots of the jerk system (4) for different values of a when we fix $b = 0.2$

Next, we fix $b = 0.2$, the initial state as $X(0) = (0.2, 0.2, 0.2)$ and vary a in $[2,3]$.

From the bifurcation diagram given in Figure 4, we see that new chaotic jerk system (4) exhibits chaos, period, reversed period-doubling route, as well as several periodic windows. Note that there is a large period-3 periodic window.

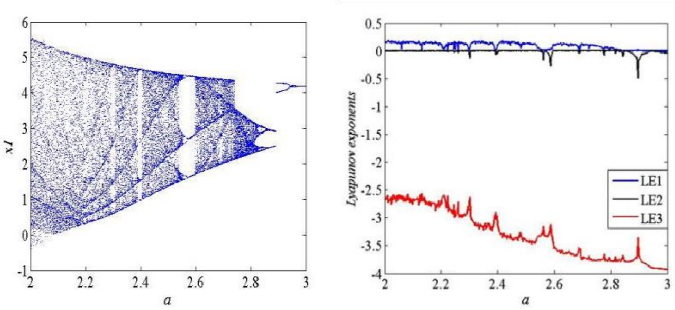


Figure 4. Bifurcation diagram and Lyapunov exponents for the new chaotic jerk system (4) when we fix $b = 0.2$ and vary a in the interval $[2,3]$

Next, we fix $a = 2$, initial state as $X(0) = (0.2, 0.2, 0.2)$ and vary b in $[0.2, 0.45]$. The corresponding bifurcation diagram and Lyapunov exponents for the new chaotic jerk system (4) are shown in Figure 5. Obviously, from the bifurcation diagram, one can get that the system shows a reversed period-doubling route to chaos in the whole region. Note that there is a large period-3 periodic window.

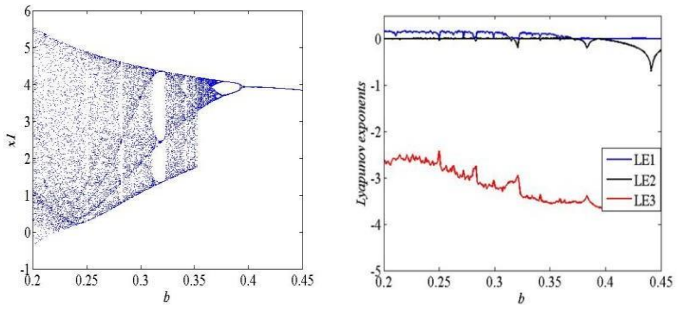


Figure 5. Bifurcation diagram and Lyapunov exponents for the new chaotic jerk system (4) when we fix $a = 2$ and vary b in the interval $[0.2, 0.45]$

Next, we describe our results for coexisting attractors. We note that the blue color denotes the trajectory of the new chaotic jerk system (4) starting from $X_0 = (0.2, 0.2, 0.2)$ and the red color denotes the trajectory of the new chaotic jerk system (4) starting from $Y_0 = (-0.2, -0.2, -0.2)$.

We fix $b = 0.2$ and vary a in the interval $[2,3]$. As can be seen from the bifurcation diagram given in Figure 6, there exists coexisting attractors in the region of $[2.75, 2.9]$.

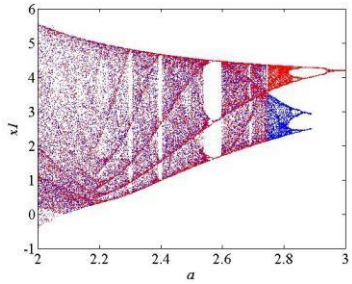
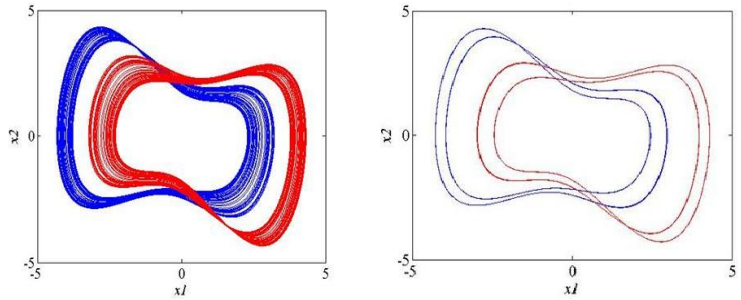


Figure 6. Bifurcation diagram for the new chaotic jerk system (4) when we fix $b = 0.2$ and vary a in the interval $[2,3]$.



(a) $a = 2.8, b = 0.2$ (b) $a = 2.86, b = 0.2$

Figure 7. (a) When $a = 2.8$, coexisting chaotic attractors, and (b) when $a = 2.86$, coexisting period-2 attractors for the new chaotic jerk system (4). We fix $b = 0.2$.

4. Backstepping-Based Global Chaos Synchronization of the Chaotic Jerk Systems

In this section, we use backstepping control method to achieve global chaos synchronization of the new chaotic jerk systems.

As the master system for the synchronization, we consider the new chaotic jerk system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -x_1 - ax_3 - bx_1^2 x_3 + x_1 x_2^2 - x_1^3 \end{cases} \tag{11}$$

In (11), x_1, x_2, x_3 are the states and a, b are unknown state parameters.

As the slave system for the synchronization, we consider the new chaotic jerk system

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = -y_1 - ay_3 - by_1^2 y_3 + y_1 y_2^2 - y_1^3 + u \end{cases} \tag{12}$$

In (12), y_1, y_2, y_3 are the states and u is a backstepping control to be designed.

The synchronization error between the jerk systems (11) and (12) can be classified as follows:

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \quad e_3 = y_3 - x_3 \tag{13}$$

We find the error dynamics as follows:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = -e_1 - ae_3 - b(y_1^2 y_3 - x_1^2 x_3) + y_1 y_2^2 - x_1 x_2^2 - y_1^3 + x_1^3 + u \end{cases} \tag{14}$$

We denote $A(t), B(t)$ as estimates for the unknown parameters a, b, respectively.

The error between the parameters and their estimates is defined as follows:

$$e_a(t) = a - A(t), \quad e_b(t) = b - B(t) \tag{15}$$

It is easy to see that

$$\begin{cases} \dot{e}_a = -\dot{A}(t) \\ \dot{e}_b = -\dot{B}(t) \end{cases} \tag{16}$$

Using adaptive backstepping control, we establish a key result of this section.

Theorem 1. The master and slave chaotic jerk systems represented by (11) and (12) with unknown parameters a and b are globally and exponentially synchronized by means of the adaptive backstepping controller using estimates $A(t)$ and $B(t)$ given by

$$u = -2e_1 - 5e_2 - [3 - A(t)]e_3 + B(t)(y_1^2 y_3 - x_1^2 x_3) - y_1 y_2^2 + x_1 x_2^2 + y_1^3 - x_1^3 - K\sigma_3 \tag{17}$$

where $K > 0$ is a gain constant,

$$\sigma_3 = 2e_1 + 2e_2 + e_3 \tag{18}$$

and the update law for the estimates $A(t), B(t)$ is given by

$$\begin{cases} \dot{A} = -\sigma_3 e_3 \\ \dot{B} = -\sigma_3 \left(y_1^2 y_3 - x_1^2 x_3 \right) \end{cases} \tag{19}$$

Proof. The result is proved via backstepping control method, which is a recursive procedure in Lyapunov stability theory. We start with the Lyapunov function

$$V_1(\sigma_1) = 0.5\ \sigma_1^2 \tag{20}$$

where $\sigma_1 = e_1$.

Differentiating V_1 along the error dynamics (14), we get

$$\dot{V}_1 = \sigma_1 \dot{\sigma}_1 = e_1 \dot{e}_1 = -e_1^2 + e_1 \left(e_1 + e_2 \right) \tag{21}$$

We define

$$\sigma_2 = e_1 + e_2 \tag{22}$$

Using Eq. (22), we can simplify Eq. (21) as

$$\dot{V}_1 = -\sigma_1^2 + \sigma_1 \sigma_2 \tag{23}$$

Next, we define the Lyapunov function

$$V_2 \left(\sigma_1, \sigma_2 \right) = V_1 (\sigma_1) + 0.5 \sigma_2^2 = 0.5 \left(\sigma_1^2 + \sigma_2^2 \right) \tag{24}$$

Differentiating V_2 along the error dynamics (14), we get

$$\dot{V}_2 = -\sigma_1^2 - \sigma_2^2 + \sigma_2 \left(2e_1 + 2e_2 + e_3 \right) \tag{25}$$

We define

$$\sigma_3 = 2e_1 + 2e_2 + e_3 \tag{26}$$

Using (26), we can express (25) as

$$\dot{V}_2 = -\sigma_1^2 - \sigma_2^2 + \sigma_2 \sigma_3 \tag{27}$$

To simplify the notation, we set $\sigma = (\sigma_1, \sigma_2, \sigma_3)$.

Finally, we define the quadratic Lyapunov function

$$V(\sigma, e_a, e_b) = 0.5 \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 \right) + 0.5 \left(e_a^2 + e_b^2 \right) \tag{28}$$

It is evident that V is a positive definite function on R^5.

Differentiating V along the error dynamics (14) and (16), we get

$$\dot{V} = -\sigma_1^2 - \sigma_2^2 - \sigma_3^2 + \sigma_3 T - e_a \dot{A} - e_b \dot{B} \tag{29}$$

where

$$T = \dot{\sigma}_3 + \sigma_2 + \dot{\sigma}_3 = \dot{\sigma}_3 + \sigma_2 + \left(2\dot{e}_1 + 2\dot{e}_2 + e_3 \right) \tag{30}$$

A simple calculation shows that

$$T = 2e_1 + 5e_2 + (3-a)e_3 - b \left(y_1^2 y_3 - x_1^2 x_3 \right) + y_1 y_2^2 - x_1 x_2^2 - y_1^3 + x_1^3 + u \tag{31}$$

Substituting the value of u from Eq. (17) into Eq. (31), we obtain

$$T = -[a - A(t)]e_3 - [b - B(t)] \left(y_1^2 y_3 - x_1^2 x_3 \right) - K\sigma_3 \tag{32}$$

We can simplify Eq. (32) by using the definitions given in Eq. (15) as follows:

$$T = -e_a e_3 - e_b \left(y_1^2 y_3 - x_1^2 x_3 \right) - K\sigma_3 \tag{33}$$

Substituting the value of T from Eq. (33) into Eq. (29), we get

$$\dot{V} = -\sigma_1^2 - \sigma_2^2 - (1+K)\sigma_3^2 + e_a(-\sigma_3 e_3 - \dot{A}) + e_b \left[-\sigma_3 \left(y_1^2 y_3 - x_1^2 x_3 \right) - \dot{B} \right] \tag{34}$$

Implementing the parameter update law (19) into Eq. (34), we get

$$\dot{V} = -\sigma_1^2 - \sigma_2^2 - (1+K)\sigma_3^2 \tag{35}$$

which is a negative semi-definite function on R^5.

Thus, by Barbalat's lemma [23] in Lyapunov stability theory, we conclude that $\sigma(t) \to 0$ exponentially as $t \to \infty$. Hence, it follows that $e(t) \to 0$ as $t \to \infty$.

This completes the proof.

For numerical plots, we take the constants (a,b) as in the chaotic case, *viz.* $(a,b) = (2, 0.2)$.

We take the gain as $K = 10$.

The initial conditions of the master jerk system (11) are picked as

$$x_1(0) = 2.6, \quad x_2(0) = 0.3, \quad x_3(0) = 1.2 \tag{36}$$

The initial conditions of the slave jerk system (12) are taken as

$$y_1(0) = 1.7, \quad y_2(0) = 3.8, \quad y_3(0) = 4.9 \tag{37}$$

The initial conditions of the parameter estimates are taken as

$$A(0) = 2.8, \; B(0) = 7.4 \tag{38}$$

Figure 8 shows the global chaos synchronization of the chaotic jerk systems (11) and (12).

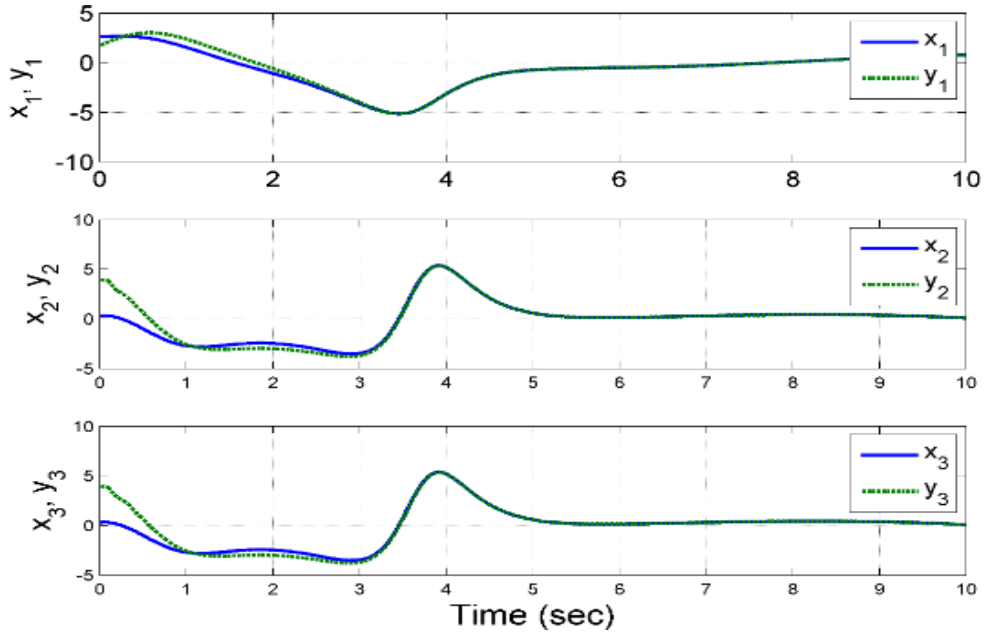


Figure 8 Synchronization of the chaotic jerk systems (11) and (12)

5. Circuit Implementation of the New Chaotic Jerk System

In this work, we describe a realization of theoretical jerk model (4) by using electronic components. As shown in Figure 9, the circuit includes three op-amp integrator circuits based on three operational amplifiers (U1A, U2A, U3A), three inverting amplifiers (U5A, U6A, U7A) which are implemented with the operational amplifier TL082CD and five multipliers by using AD633JN.

The circuital equations of the designed jerk circuit are given by

$$\begin{cases} \dot{x}_1 = \dfrac{1}{C_1 R_1} x_2 \\ \dot{x}_2 = \dfrac{1}{C_2 R_2} x_3 \\ \dot{x}_3 = -\dfrac{1}{C_3 R_3} x_1 - \dfrac{1}{C_3 R_4} x_3 - \dfrac{1}{100 C_3 R_5} x_1^2 x_3 + \dfrac{1}{100 C_3 R_6} x_1 x_2^2 - \dfrac{1}{100 C_3 R_7} x_1^3 \end{cases} \tag{39}$$

where x , y , and z are the voltages across the capacitors C_1, C_2 and C_3, respectively. Equations (39) match Eqs. (1) when the circuit components are selected as follows: R_4 = 50 kfi, R_5 = 5 kfi, $R_6 = R_7$ = 1 kfi, $R_1 = R_2 = R_3 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13}$ = 100 kfi, $C_1 = C_2 = C_3$ = 1 nF. MultiSIM phase portraits of the circuit are represented in Figure 10. Once more a very good qualitative agreement can be observed between numerical simulations (see Figure 1) and MultiSIM results (see Figure 10).

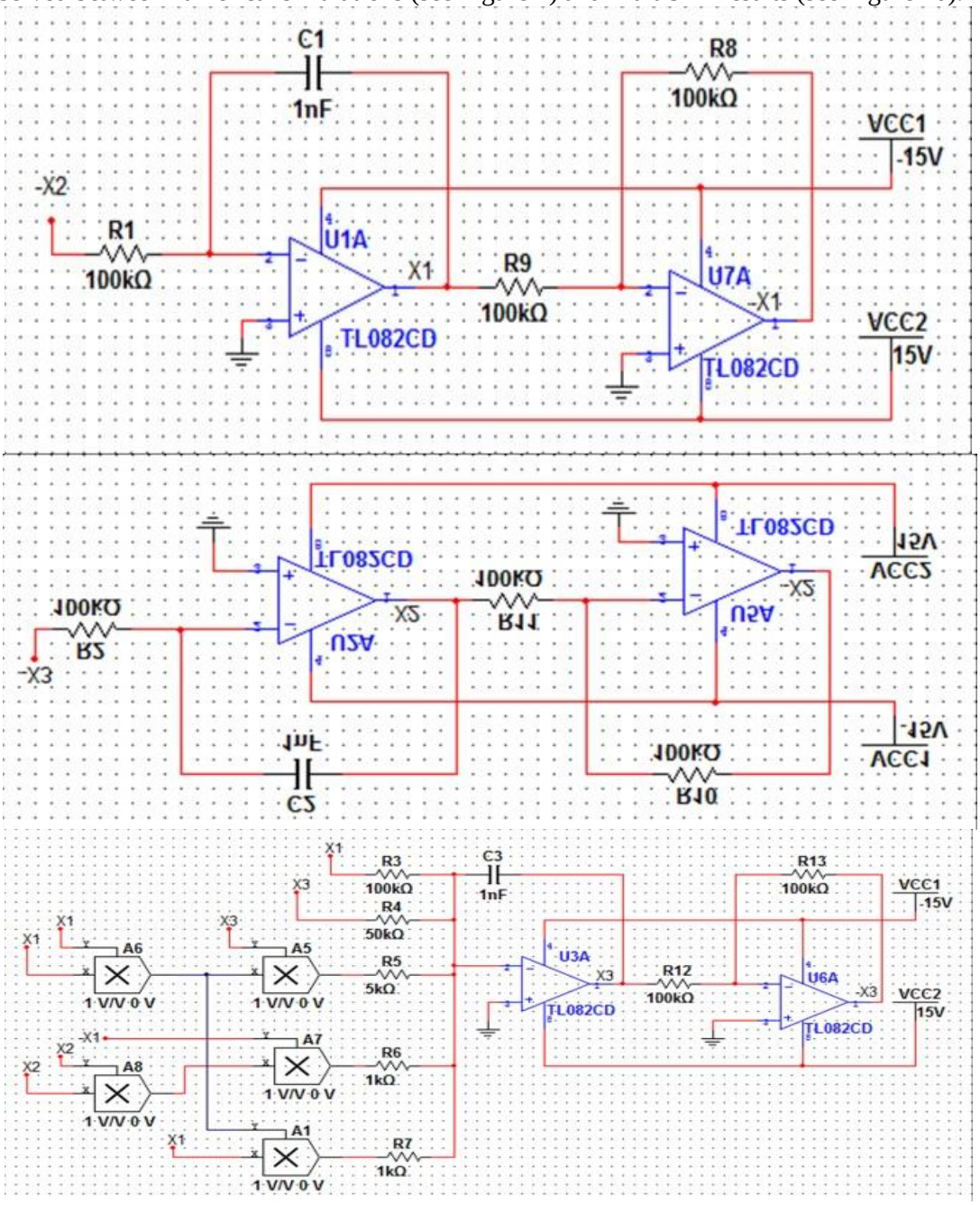
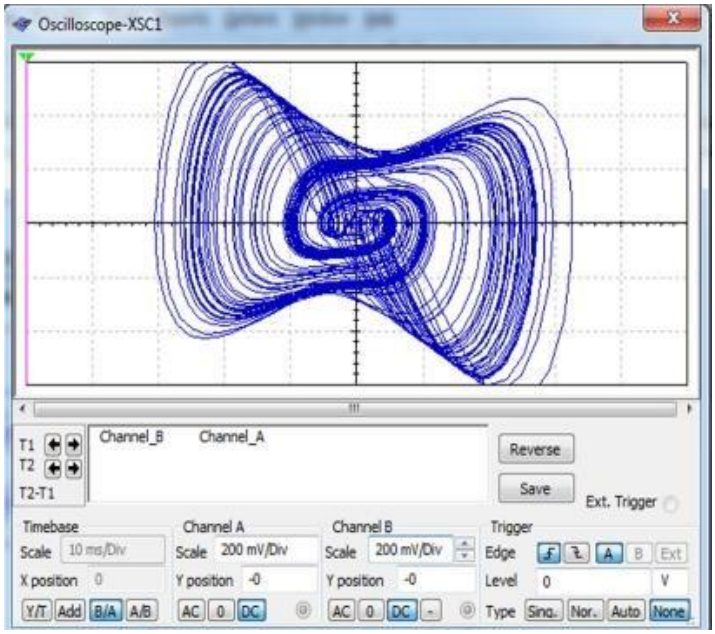
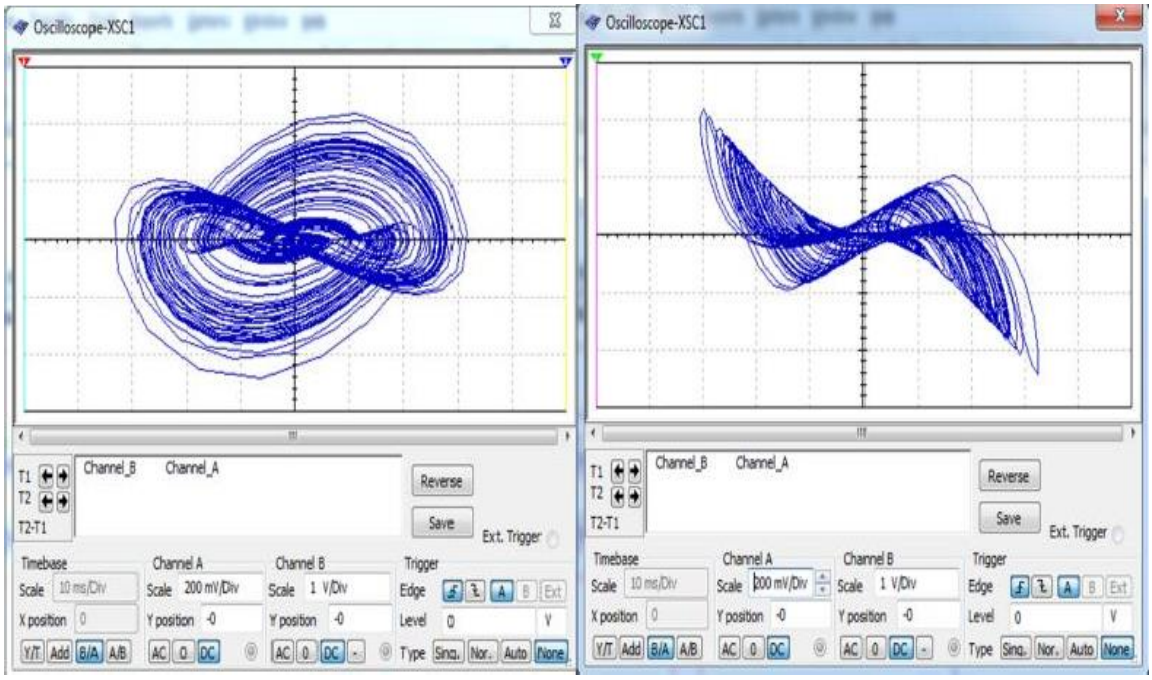


Figure 9. The electronic circuit schematic of new chaotic jerk system



(a)



(b) (c)

Figure 10. MultiSIM chaotic attractors of the new chaotic jerk system (4) (a) x_1- x_2 plane (b) x_2- x_3 plane and (c) x_1-x_3 plane.

6. Conclusions

A new chaotic jerk system with three cubic nonlinearities is proposed in this paper. The dynamical properties of the new jerk system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, symmetry, dissipativity, etc. Also, a detailed dynamical analysis of the jerk system was done using bifurcation diagram and Lyapunov exponents, and we exhibited coexisting chaotic attractors for the new jerk system. The adaptive backstepping-based synchronization of the

new chaotic jerk system with itself was designed. Furthermore, an electronic circuit realization of the new jerk system was shown to confirm the feasibility of the jerk system.

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A Chaotic Jerk System with Three Cubic Nonlinearities, Dynamical Analysis, Adaptive Chaos Synchronization and Circuit Simulation

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A Chaotic Jerk System with Three Cubic Nonlinearities, Dynamical Analysis, Adaptive Chaos Synchronization and Circuit Simulation

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Abstract. A 3-D new chaotic jerk system with three cubic nonlinearities is proposed in this paper. The dynamical properties of the new jerk system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, symmetry, dissipativity, etc. Also, a detailed dynamical analysis of the jerk system has been carried out with bifurcation diagram and Lyapunov exponents. As an engineering application, adaptive synchronization of the new chaotic jerk system with itself is designed via backstepping control method. Furthermore, an electronic circuit realization of the new chaotic jerk system is presented in detail to confirm the feasibility of the theoretical chaotic jerk model.

1. Introduction

Chaos theory deals with chaotic systems which are nonlinear dynamical systems exhibiting high sensitivity to small changes in initial conditions [1-2]. Chaotic systems are very useful in many applications in science and engineering such as temperature model [3], biology [4], physics [5], cellular neural networks [6], satellite [7], robotics [8], encryption [9], finance systems [10], circuits [11-12], etc.

In physics, a jerk ODE can be written as the third order dynamics

$$\frac{d^3x}{dt^3} = \varphi\left(x, \frac{dx}{dt}, \frac{d^2x}{dt^2}\right) \tag{1}$$

In (1), $x(t)$ stands for the displacement, $\frac{dx}{dt}$ the velocity, $\frac{d^2x}{dt^2}$ the acceleration and $\frac{d^3x}{dt^3}$ the jerk.

Thus, we call the ODE (1) as the jerk differential equation.

For qualitative analysis, it is convenient to express the third-order ODE (1) in a system form. This is achieved by defining the following phase variables:

$$\begin{cases} x_1(t) = x(t) \\ x_2(t) = \dot{x}(t) \\ x_3(t) = \ddot{x}(t) \end{cases} \tag{2}$$

Thus, the phase variables in (2) can be viewed as the displacement, velocity and acceleration, respectively. Using them, we can express the jerk differential equation (1) as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = \varphi(x_1, x_2, x_3) \end{cases} \tag{3}$$

Many jerk systems have been found in the chaos literature [13-17]. Jerk systems have important applications in mechanical engineering [1-2]. Some famous jerk systems are Sprott systems [13], Li system [14], Elsonbaty system [15], Coullet system [16], Vaidyanathan systems [17], etc.

In this research paper, we report the finding of a new chaotic jerk system with three cubic nonlinearities. We describe the phase plots of the jerk system and do a rigorous dynamic analysis by finding bifurcation diagrams, Lyapunov exponents, Kaplan-Yorke dimension, symmetry analysis, etc.

As a control application, we derive new results for the adaptive synchronization of the new chaotic jerk system with itself with unknown parameters. Synchronization of chaotic systems deals with the control problem of finding suitable feedback control laws so as to asymptotically synchronize the respective trajectories of a pair of chaotic systems called as master and slave systems. We use backstepping control method for achieving global chaos synchronization of the new chaotic jerk system with itself. Backstepping control method is a recursive procedure used for stabilizing nonlinear dynamical systems.

Section 2 describes the new chaotic jerk system, its phase plots and Lyapunov exponents. Section 3 describes the dynamic analysis of the new chaotic jerk system. Section 4 describes the backstepping-based adaptive synchronization of the new chaotic jerk system with itself. Furthermore, an electronic circuit realization of the new chaotic system is presented in detail in Section 5. The circuit experimental results of the new chaotic jerk system in Section 5 agreement with the numerical simulations via MATLAB obtained in Section 2. Section 6 contains the main conclusions.

2. A new chaotic jerk system with three cubic nonlinearities

In this work, we report a new 3-D chaotic jerk system given by the dynamics

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -x_1 - ax_3 - bx_1^2 x_3 + x_1 x_2^2 - x_1^3 \end{cases} \tag{4}$$

where x_1, x_2, x_3 are state variables and a, b are positive constants.

In this paper, we show that the jerk system (1) is *chaotic* for the parameter values

$$a = 2, \quad b = 0.2 \tag{5}$$

For numerical simulations, we take the initial values of the jerk system (4) as $X(0) = (0.2, 0.2, 0.2)$. Figure 1 shows the phase portraits of the strange attractor of the new chaotic jerk system (4) for $(a, b) = (2, 0.2)$ and initial conditions $X(0) = (0.2, 0.2, 0.2)$. Figure 1 (a) shows the 3-D phase portrait of the new chaotic jerk system (4). Figures 1 (b)-(c) show the projections of the new chaotic system (4) in (x_1, x_2), (x_2, x_3) and (x_1, x_3) coordinate planes, respectively.

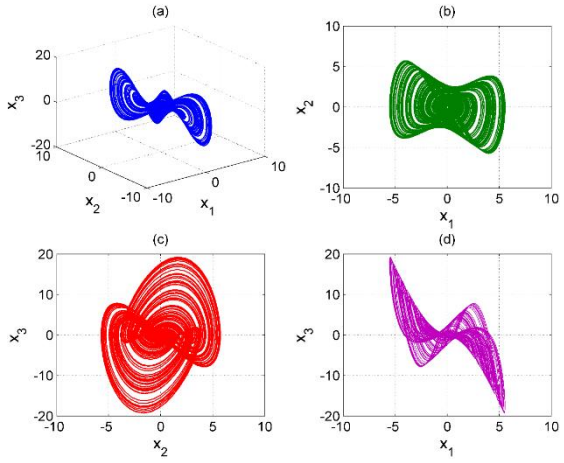


Figure 1. Plots of the chaotic jerk system (4) for $(a,b)=(2,0.2)$ and $X(0)=(0.2,0.2,0.2)$

For the rest of this section, we take the values of the parameters a and b as in the chaotic case (5), *i.e.* $(a,b)=(2,0.2)$

The equilibrium points of the new chaotic jerk system (4) are obtained by solving the system of equations

$$x_2 = 0 \tag{6a}$$

$$x_3 = 0 \tag{6b}$$

$$-x_1 - ax_3 - bx_1^2 x_3 + x_1 x_2^2 - x_1^3 = 0 \tag{6c}$$

From (6a) and (6b), we deduce that $x_2 = x_3 = 0.$

Substituting these in (6c), we obtain $-x_1\left(1+x_1^2\right)=0.$ This gives $x_1 = 0.$

Hence, $E_0 = (0,0,0)$ is the unique equilibrium of the chaotic jerk system (4).

The Jacobian matrix of the new jerk system (4) at $E_0 = (0,0,0)$ is obtained as

$$J = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -2 \end{bmatrix} \tag{7}$$

The Jacobian matrix J has the spectral values $-2.2056,\ 0.1028 \pm 0.6655i.$

This shows that the equilibrium point E_0 is a saddle-focus and unstable.

We note that the new chaotic system (4) is invariant under the coordinates transformation

$$\left(x_1, x_2, x_3\right) \mapsto \left(-x_1, -x_2, -x_3\right) \tag{8}$$

for all values of the parameters. This shows that the new chaotic system (4) has point-reflection symmetry about the equilibrium $E_0 = (0,0,0).$

For the parameter values as in the chaotic case (5) and the initial state $X(0)=(0.2,0.2,0.2),$ the Lyapunov exponents of the new jerk system (4) are determined using Wolf's algorithm as

$$LE_1 = 0.1680,\ LE_2 = 0,\ LE_3 = -2.6705 \tag{9}$$

The jerk system (4) is chaotic since $LE_1 = 0.1680 > 0.$ Thus, the system (4) exhibits a self-excited strange chaotic attractor. Also, we note that the sum of the Lyapunov exponents in (9) is negative. This shows that the jerk system (4) is dissipative.

The Kaplan-Yorke dimension of the jerk system (4) is determined as

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.0629, \tag{10}$$

which indicates the high complexity of the chaotic jerk system (4).
Figure 2 shows the Lyapunov exponents of the new chaotic jerk system (4) with self-excited, strange chaotic attractor.

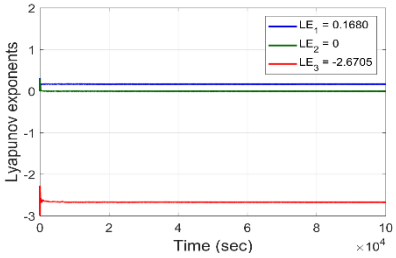
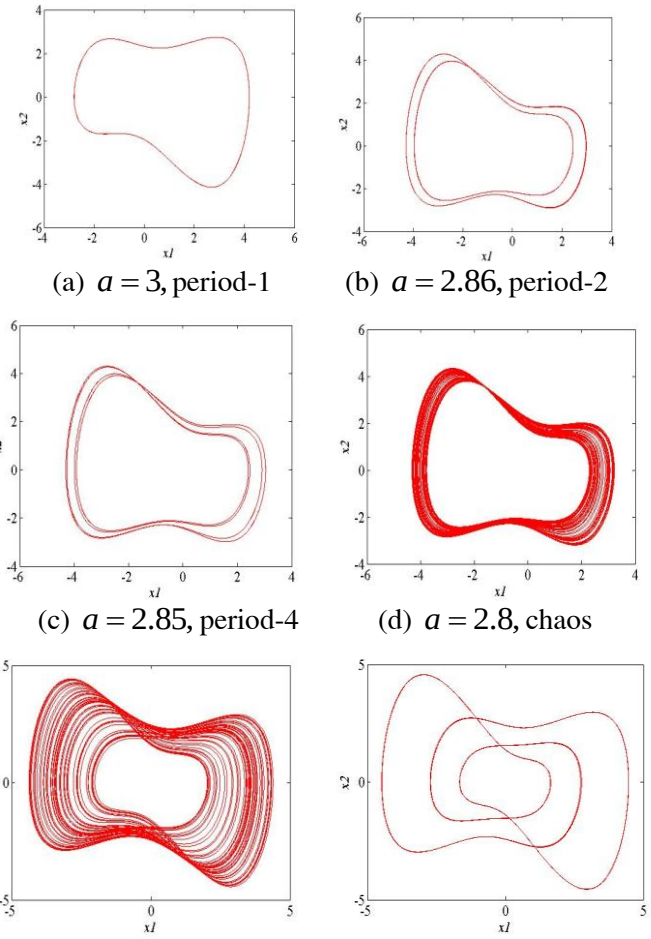


Figure 2. Lyapunov exponents of the new chaotic jerk system (4) for $(a,b) = (2, 0.2)$

3. Bifurcation Analysis for the New Chaotic Jerk System

In this section, we describe a bifurcation analysis for the new chaotic jerk system (4) introduced in Section 2. Bifurcation analysis is an important topic for understanding chaotic systems [18-22].

Some sample results are plotted to verify the reversed period-doubling route to chaos in Figure 3.



(a) $a = 3$, period-1 (b) $a = 2.86$, period-2

(c) $a = 2.85$, period-4 (d) $a = 2.8$, chaos

(e) $a = 2.7$, chaos (f) $a = 2.58$, period-3

Figure 3. Some sample plots of the jerk system (4) for different values of a when we fix $b = 0.2$

Next, we fix $b = 0.2,$ the initial state as $X(0) = (0.2, 0.2, 0.2)$ and vary a in $[2, 3].$
From the bifurcation diagram given in Figure 4, we see that new chaotic jerk system (4) exhibits chaos, period, reversed period-doubling route, as well as several periodic windows. Note that there is a large period-3 periodic window.

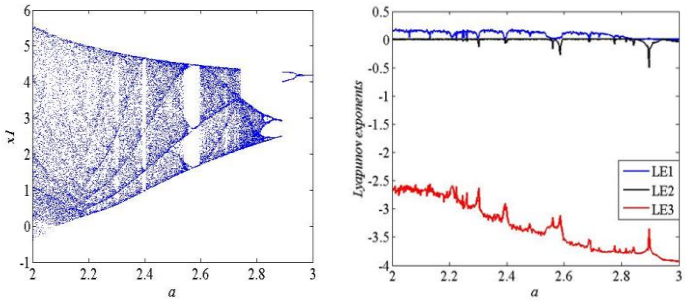


Figure 4. Bifurcation diagram and Lyapunov exponents for the new chaotic jerk system (4) when we fix $b = 0.2$ and vary a in the interval $[2, 3]$

Next, we fix $a = 2,$ initial state as $X(0) = (0.2, 0.2, 0.2)$ and vary b in $[0.2, 0.45].$ The corresponding bifurcation diagram and Lyapunov exponents for the new chaotic jerk system (4) are shown in Figure 5. Obviously, from the bifurcation diagram, one can get that the system shows a reversed period-doubling route to chaos in the whole region. Note that there is a large period-3 periodic window.

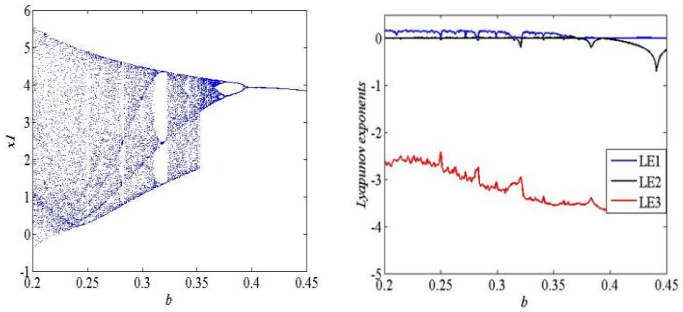


Figure 5. Bifurcation diagram and Lyapunov exponents for the new chaotic jerk system (4) when we fix $a = 2$ and vary b in the interval $[0.2, 0.45]$

Next, we describe our results for coexisting attractors. We note that the blue color denotes the trajectory of the new chaotic jerk system (4) starting from $X_0 = (0.2, 0.2, 0.2)$ and the red color denotes the trajectory of the new chaotic jerk system (4) starting from $Y_0 = (-0.2, -0.2, -0.2).$

We fix $b = 0.2$ and vary a in the interval $[2, 3].$ As can be seen from the bifurcation diagram given in Figure 6, there exists coexisting attractors in the region of $[2.75, 2.9].$

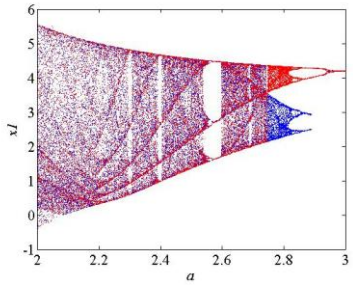
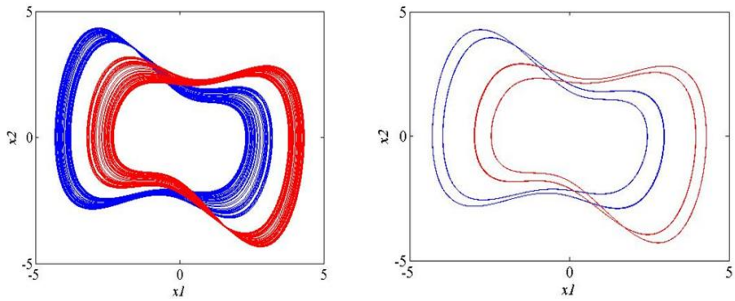


Figure 6. Bifurcation diagram for the new chaotic jerk system (4) when we fix $b = 0.2$ and vary a in the interval $[2, 3].$



(a) $a = 2.8, b = 0.2$ (b) $a = 2.86, b = 0.2$

Figure 7. (a) When $a = 2.8,$ coexisting chaotic attractors, and (b) when $a = 2.86,$ coexisting period-2 attractors for the new chaotic jerk system (4). We fix $b = 0.2.$

4. Backstepping-Based Global Chaos Synchronization of the Chaotic Jerk Systems

In this section, we use backstepping control method to achieve global chaos synchronization of the new chaotic jerk systems.

As the master system for the synchronization, we consider the new chaotic jerk system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -x_1 - ax_3 - bx_1^2 x_3 + x_1 x_2^2 - x_1^3 \end{cases} \tag{11}$$

In (11), x_1, x_2, x_3 are the states and a, b are unknown state parameters.

As the slave system for the synchronization, we consider the new chaotic jerk system

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = -y_1 - ay_3 - by_1^2 y_3 + y_1 y_2^2 - y_1^3 + u \end{cases} \tag{12}$$

In (12), y_1, y_2, y_3 are the states and u is a backstepping control to be designed.

The synchronization error between the jerk systems (11) and (12) can be classified as follows:

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \quad e_3 = y_3 - x_3 \tag{13}$$

We find the error dynamics as follows:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = -e_1 - ae_3 - b(y_1^2 y_3 - x_1^2 x_3) + y_1 y_2^2 - x_1 x_2^2 - y_1^3 + x_1^3 + u \end{cases} \tag{14}$$

We denote $A(t), B(t)$ as estimates for the unknown parameters $a, b,$ respectively.

The error between the parameters and their estimates is defined as follows:

$$e_a(t) = a - A(t), \quad e_b(t) = b - B(t) \tag{15}$$

It is easy to see that

$$\begin{cases} \dot{e}_a = -\dot{A}(t) \\ \dot{e}_b = -\dot{B}(t) \end{cases} \tag{16}$$

Using adaptive backstepping control, we establish a key result of this section.

Theorem 1. The master and slave chaotic jerk systems represented by (11) and (12) with unknown parameters a and b are globally and exponentially synchronized by means of the adaptive backstepping controller using estimates $A(t)$ and $B(t)$ given by

$$u = -2e_1 - 5e_2 - [3 - A(t)]e_3 + B(t)\left(y_1^2 y_3 - x_1^2 x_3\right) - y_1 y_2^2 + x_1 x_2^2 + y_1^3 - x_1^3 - K\sigma_3 \tag{17}$$

where $K > 0$ is a gain constant,

$$\sigma_3 = 2e_1 + 2e_2 + e_3 \tag{18}$$

and the update law for the estimates $A(t), B(t)$ is given by

$$\begin{cases} \dot{A} = -\sigma_3 e_3 \\ \dot{B} = -\sigma_3 \left(y_1^2 y_3 - x_1^2 x_3 \right) \end{cases} \tag{19}$$

Proof. The result is proved via backstepping control method, which is a recursive procedure in Lyapunov stability theory. We start with the Lyapunov function

$$V_1(\sigma_1) = 0.5\ \sigma_1^2 \tag{20}$$

where $\sigma_1 = e_1$.

Differentiating V_1 along the error dynamics (14), we get

$$\dot{V}_1 = \sigma_1 \dot{\sigma}_1 = e_1 e_2 = -e_1^2 + e_1 \left(e_1 + e_2 \right) \tag{21}$$

We define

$$\sigma_2 = e_1 + e_2 \tag{22}$$

Using Eq. (22), we can simplify Eq. (21) as

$$\dot{V}_1 = -\sigma_1^2 + \sigma_1 \sigma_2 \tag{23}$$

Next, we define the Lyapunov function

$$V_2 \left(\sigma_1, \sigma_2 \right) = V_1(\sigma_1) + 0.5\sigma_2^2 = 0.5\left(\sigma_1^2 + \sigma_2^2 \right) \tag{24}$$

Differentiating V_2 along the error dynamics (14), we get

$$\dot{V}_2 = -\sigma_1^2 - \sigma_2^2 + \sigma_2 \left(2e_1 + 2e_2 + e_3 \right) \tag{25}$$

We define

$$\sigma_3 = 2e_1 + 2e_2 + e_3 \tag{26}$$

Using (26), we can express (25) as

$$\dot{V}_2 = -\sigma_1^2 - \sigma_2^2 + \sigma_2 \sigma_3 \tag{27}$$

To simplify the notation, we set $\sigma = (\sigma_1, \sigma_2, \sigma_3)$.

Finally, we define the quadratic Lyapunov function

$$V(\sigma, e_a, e_b) = 0.5\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 \right) + 0.5\left(e_a^2 + e_b^2 \right) \tag{28}$$

It is evident that V is a positive definite function on R^5.

Differentiating V along the error dynamics (14) and (16), we get

$$\dot{V} = -\sigma_1^2 - \sigma_2^2 - \sigma_3^2 + \sigma_3 T - e_a \dot{A} - e_b \dot{B} \tag{29}$$

where

$$T = \sigma_3 + \sigma_2 + \dot{\sigma}_3 = \sigma_3 + \sigma_2 + \left(2\dot{e}_1 + 2\dot{e}_2 + e_3 \right) \tag{30}$$

A simple calculation shows that

$$T = 2e_1 + 5e_2 + (3 - a)e_3 - b\left(y_1^2 y_3 - x_1^2 x_3 \right) + y_1 y_2^2 - x_1 x_2^2 - y_1^3 + x_1^3 + u \tag{31}$$

Substituting the value of u from Eq. (17) into Eq. (31), we obtain

$$T = -[a - A(t)]e_3 - [b - B(t)]\left(y_1^2 y_3 - x_1^2 x_3 \right) - K\sigma_3 \tag{32}$$

We can simplify Eq. (32) by using the definitions given in Eq. (15) as follows:

$$T = -e_a e_3 - e_b \left(y_1^2 y_3 - x_1^2 x_3 \right) - K\sigma_3 \tag{33}$$

Substituting the value of T from Eq. (33) into Eq. (29), we get

$$\dot{V} = -\sigma_1^2 - \sigma_2^2 - (1 + K)\sigma_3^2 + e_a(-\sigma_3 e_3 - \dot{A}) + e_b\left[-\sigma_3 \left(y_1^2 y_3 - x_1^2 x_3 \right) - \dot{B} \right] \tag{34}$$

Implementing the parameter update law (19) into Eq. (34), we get

$$\dot{V} = -\sigma_1^2 - \sigma_2^2 - (1+K)\sigma_3^2 \tag{35}$$

which is a negative semi-definite function on R^5.

Thus, by Barbalat's lemma [23] in Lyapunov stability theory, we conclude that $\sigma(t) \to 0$ exponentially as $t \to \infty$. Hence, it follows that $e(t) \to 0$ as $t \to \infty$.

This completes the proof.

For numerical plots, we take the constants (a,b) as in the chaotic case, *viz.* $(a,b) = (2, 0.2)$.

We take the gain as $K = 10$.

The initial conditions of the master jerk system (11) are picked as

$$x_1(0) = 2.6, \quad x_2(0) = 0.3, \quad x_3(0) = 1.2 \tag{36}$$

The initial conditions of the slave jerk system (12) are taken as

$$y_1(0) = 1.7, \quad y_2(0) = 3.8, \quad y_3(0) = 4.9 \tag{37}$$

The initial conditions of the parameter estimates are taken as

$$A(0) = 2.8, \quad B(0) = 7.4 \tag{38}$$

Figure 8 shows the global chaos synchronization of the chaotic jerk systems (11) and (12).

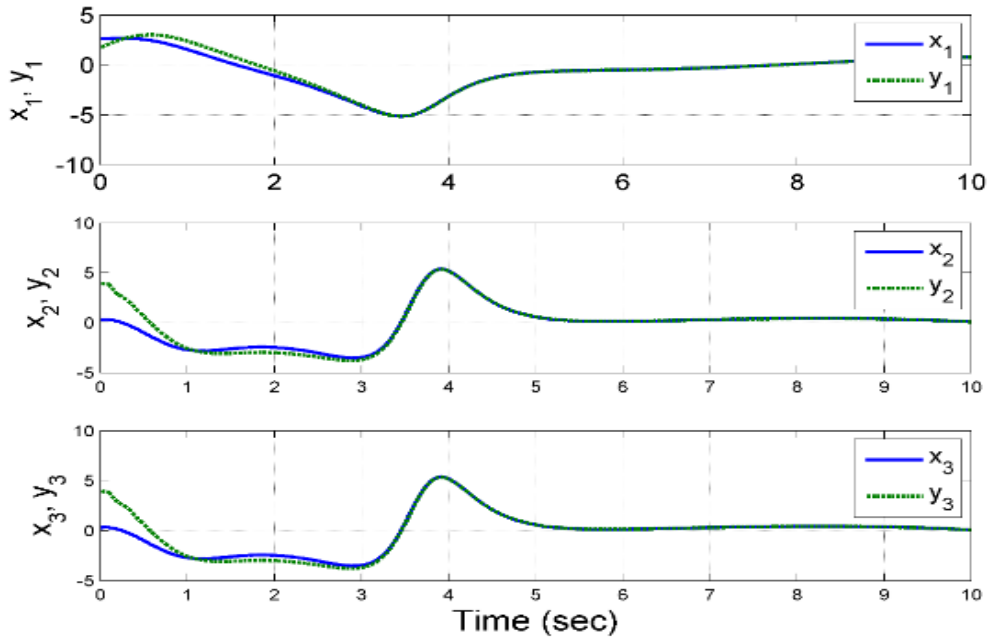


Figure 8 Synchronization of the chaotic jerk systems (11) and (12)

5. Circuit Implementation of the New Chaotic Jerk System

In this work, we describe a realization of theoretical jerk model (4) by using electronic components. As shown in Figure 9, the circuit includes three op-amp integrator circuits based on three operational amplifiers (U1A, U2A, U3A), three inverting ampli□ers (U5A, U6A, U7A) which are implemented with the operational ampli□er TL082CD and □ve multipliers by using AD633JN.

The circuital equations of the designed jerk circuit are given by

$$\begin{cases} \dot{x}_1 = \dfrac{1}{C_1 R_1} x_2 \\ \dot{x}_2 = \dfrac{1}{C_2 R_2} x_3 \\ \dot{x}_3 = -\dfrac{1}{C_3 R_3} x_1 - \dfrac{1}{C_3 R_4} x_3 - \dfrac{1}{100 C_3 R_5} x_1^2 x_3 + \dfrac{1}{100 C_3 R_6} x_1 x_2^2 - \dfrac{1}{100 C_3 R_7} x_1^3 \end{cases} \tag{39}$$

where x , y , and z are the voltages across the capacitors C_1, C_2 and C_3, respectively. Equations (39) match Eqs. (1) when the circuit components are selected as follows: R_4 = 50 k□, R_5 = 5 k□, $R_6 = R_7$ = 1 k□, $R_1 = R_2 = R_3 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13}$ = 100 k□, $C_1 = C_2 = C_3$ = 1 nF. MultiSIM phase portraits of the circuit are represented in Figure 10. Once more a very good qualitative agreement can be observed between numerical simulations (see Figure 1) and MultiSIM results (see Figure 10).

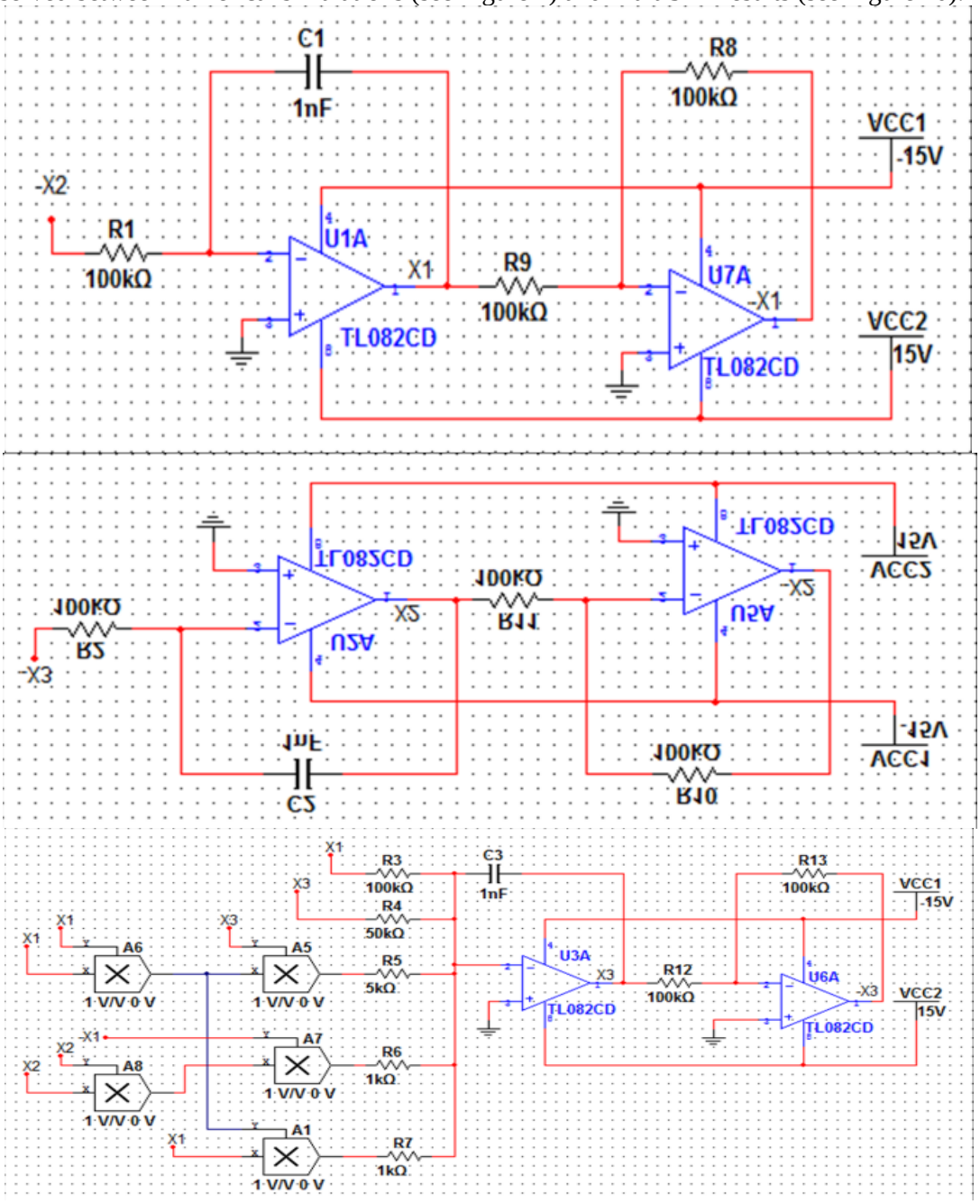
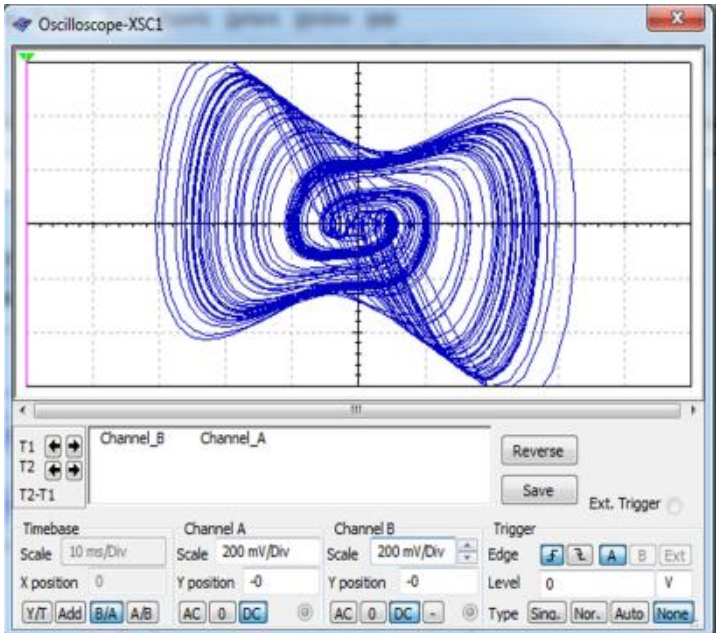
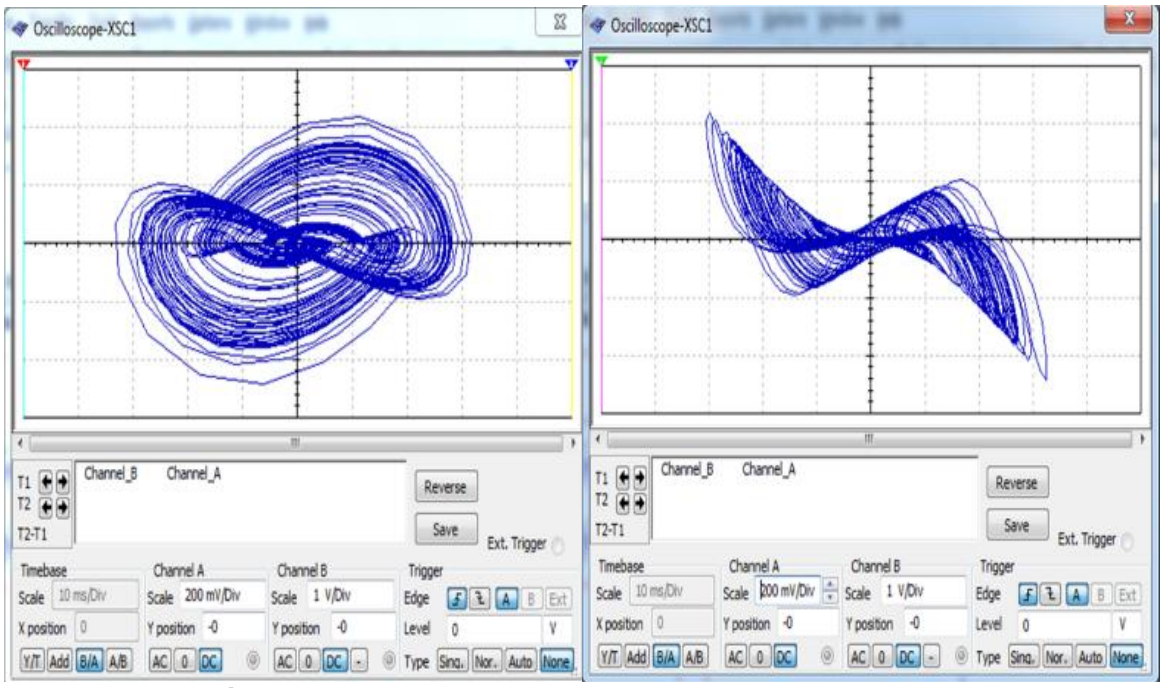


Figure 9. The electronic circuit schematic of new chaotic jerk system



(a)



(b) (c)

Figure 10. MultiSIM chaotic attractors of the new chaotic jerk system (4)
(a) x_1- x_2 plane (b) x_2- x_3 plane and (c) x_1-x_3 plane.

6. Conclusions

A new chaotic jerk system with three cubic nonlinearities is proposed in this paper. The dynamical properties of the new jerk system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, symmetry, dissipativity, etc. Also, a detailed dynamical analysis of the jerk system was done using bifurcation diagram and Lyapunov exponents, and we exhibited coexisting chaotic attractors for the new jerk system. The adaptive backstepping-based synchronization of the

new chaotic jerk system with itself was designed. Furthermore, an electronic circuit realization of the new jerk system was shown to confirm the feasibility of the jerk system.

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