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**A New 4-D Chaotic System with Self-Excited Two-Wing Attractor, its Dynamical Analysis and Circuit Realization**

A Sambas1 \*, S Vaidyanathan2 , S Zhang3 , Mujiarto1 , M Mamat4 , Subiyanto5 and W. S. Mada Sanjaya6

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**Abstract**

A new four-dimensional chaotic system with only two quadratic nonlinearities is proposed in this paper. It is interesting that the new chaotic system exhibits a two-wing strange attractor. The dynamical properties of the new chaotic system are described in terms of phase portraits, equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. The new chaotic system has two saddle-foci, unstable equilibrium points. Thus, the new chaotic system exhibits self-excited attractor. Also, a detailed analysis of the new chaotic system dynamics has been carried out with bifurcation diagram and Lyapunov exponents. As an engineering application, an electronic circuit realization of the new chaotic system is designed via MultiSIM to confirm the feasibility of the theoretical 4-D chaotic model. **(Approx. 116 words)**

**Keywords:** 4-D Chaotic System, Self-Excited

**Topic:** Engineering and Technology

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The 1<sup>st</sup> International Conference on Computer, Science, Engineering and Technology

Universitas Muhammadiyah Tasikmalaya/Tasikmalaya,

27-28 November 2018

Website: <http://www.iccomset.umtas.ac.id>

Email: [iccomset@umtas.ac.id](mailto:iccomset@umtas.ac.id)

Date: 25 September 2018

## Letter of Acceptance for Abstract

Dear Authors: A Sambas1 \*, S Vaidyanathan2 , S Zhang3 , Mujiarto1 , M Mamat4 , Subiyanto5 and W. S. Mada Sanjaya6

We are pleased to inform you that your abstract (ABS-226, Oral Presentation), entitled:

**"A New 4-D Chaotic System with Self-Excited Two-Wing Attractor, its Dynamical Analysis and Circuit Realization"**

has been reviewed and accepted to be presented at ICComSET 2018 conference to be held on 27-28 November 2018 in Tasikmalaya, Indonesia.

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Best regards,

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Dr. Mujiarto, S.T.,M.T.  
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Date: 11 October 2018

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**Abstract.** A new four-dimensional chaotic system with only two quadratic nonlinearities is proposed in this paper. It is interesting that the new chaotic system exhibits a two-wing strange attractor. The dynamical properties of the new chaotic system are described in terms of phase portraits, equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. The new chaotic system has two saddle-foci, unstable equilibrium points. Thus, the new chaotic system exhibits self-excited attractor. Also, a detailed analysis of the new chaotic system dynamics has been carried out with bifurcation diagram and Lyapunov exponents. As an engineering application, an electronic circuit realization of the new chaotic system is designed via MultiSIM to confirm the feasibility of the theoretical 4-D chaotic model.

## 1. Introduction

In the last few decades, much attention has been devoted to the modelling and analysis of many types of chaotic systems [1-2]. Chaotic systems are very useful in many applications in science and engineering such as weather systems [3-4], ecology [5], neurons [6-7], biology [8-10], cellular neural networks [11-12], chemical reactors [13-14], oscillators [15-20], robotics [21-24], encryption [25-30], finance systems [31-32], circuits [33-45], secure communication [46-50], etc.

In the chaos literature, there is great interest about finding chaotic systems with multi-wing attractors. In the chaos literature, there are chaotic systems with two-wing attractors [51-55], three-wing attractors [56-57] and four-wing attractors [58-60].

In this research paper, we report the finding of a new 4-D chaotic system exhibiting a two-wing strange attractor. Our chaotic system has a simple structure with only two quadratic nonlinearities. We show

that the new chaotic system has two saddle-foci, unstable equilibrium points. Thus, the new chaotic system exhibits self-excited chaotic attractor [2].

We describe the phase plots of the chaotic system and conduct a rigorous bifurcation analysis of the system. Bifurcation analysis is very useful to understand the special properties of chaotic and hyperchaotic systems [61-66]. As an engineering application, we build an electronic circuit of the new chaotic system and show that the circuit experimental results of the new chaotic system show good agreement with the numerical MATLAB simulations of the system. Circuit simulation is an important work associated with a chaotic system as this can be used for applications of chaotic systems [67-70].

Section 2 describes the new chaotic system with two-wing attractor, its phase plots and Lyapunov exponents. We show that the new chaotic system has two saddle-foci equilibrium points which are unstable. Hence, we shall deduce that the new chaotic system exhibits self-excited, two-wing, chaotic attractor. Section 3 describes the dynamic analysis of the new chaotic system using bifurcation diagrams and Lyapunov exponents. Also, an electronic circuit realization of the new chaotic system is presented in detail in Section 4. Finally, Section 5 summarizes the main results derived in this work.

## 2. A new 4-D chaotic system

In this work, we report a new 4-D chaotic system given by the dynamics

$$\begin{cases} \dot{x} = a(y-x) - w \\ \dot{y} = xz \\ \dot{z} = b - xy \\ \dot{w} = x - w \end{cases} \quad (1)$$

where  $x, y, z, w$  are state variables and  $a, b$  are positive constants.  $Y=1/4, x=1/2, w=1/2, z=1/4$

We note that the system (1) has a simple structure having only two quadratic nonlinearities.

In this paper, we show that the 4-D system (1) has a *two-wing chaotic attractor* when the system parameters take the following values

$$a = 5, \quad b = 50 \quad (2)$$

For numerical simulations and calculating the Lyapunov exponents, we take the initial values of the 4-D system (1) as  $X(0) = (0.2, 0.2, 0.2, 0.2)$ . Using Wolf's algorithm [71], the Lyapunov exponents of the 4-D system (1) are found as

$$LE_1 = 1.3182, \quad LE_2 = 0, \quad LE_3 = -1.0459, \quad LE_4 = -6.2723 \quad (3)$$

Also, the Kaplan-Yorke dimension of the 4-D chaotic system (1) is determined as follows:

$$D_{KY} = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} = 3.0434 \quad (4)$$

Thus, the 4-D chaotic system exhibits complex chaotic behavior.

Also, the system (1) is invariant under the coordinates transformation

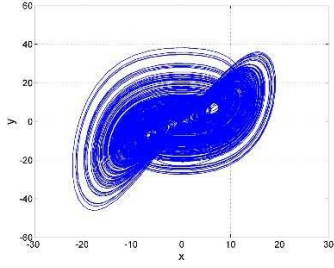
$$(x, y, z, w) \rightarrow (-x, -y, z, -w) \quad (5)$$

Hence, the chaotic system (1) has rotation symmetry about the  $z$ -axis. Every non-trivial trajectory of the system (1) has a twin trajectory because of this rotation symmetry property about the  $z$ -axis.

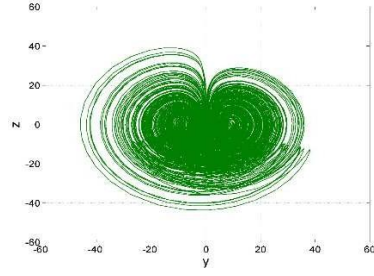
Figures 1-4 show the 2-D projections of the new chaotic system (1) in  $(x, y)$ ,  $(y, z)$ ,  $(z, w)$  and  $(x, w)$  coordinate planes, respectively.

From the phase portraits, we note that the chaotic system (1) exhibits a two-wing strange attractor.

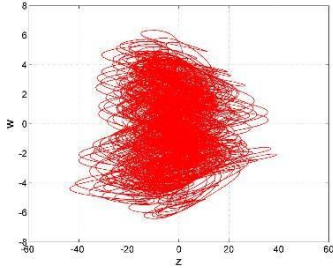




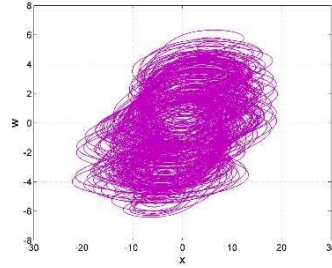
**Figure 1.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, y)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$



**Figure 2.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, y)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$



**Figure 3.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, z)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$



**Figure 4.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, w)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$

For the rest of this section, we take the values of the parameters as in the chaotic case (2), *i.e.*  $a = 5$  and  $b = 50$ .

The equilibrium points of the new chaotic system (1) are obtained by solving the system of equations

$$a(y - x) - w = 0 \quad (6a)$$

$$xz = 0 \quad (6b)$$

$$b - xy = 0 \quad (6c)$$

$$x - w = 0 \quad (6d)$$

Solving the system (6), we get two equilibrium points of the chaotic system (1) given as follows:

$$E_1 = \begin{bmatrix} 6.4550 \\ 7.7460 \\ 0 \\ 6.4550 \end{bmatrix}, E_2 = \begin{bmatrix} -6.4550 \\ -7.7460 \\ 0 \\ -6.4550 \end{bmatrix} \quad (7)$$

It is easy to verify that the equilibrium points  $E_1, E_2$  are saddle-foci, unstable, equilibrium points.

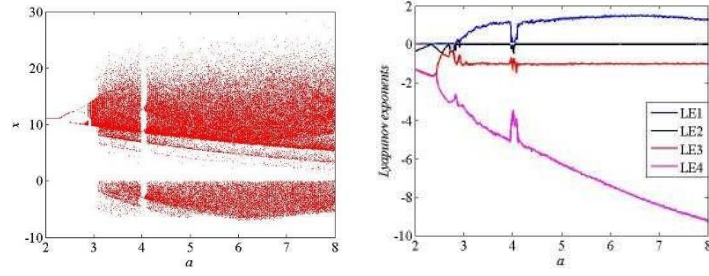
This shows that the new chaotic system (1) exhibits a self-excited, two-wing, chaotic attractor.

### 3. Bifurcation Analysis for the New Chaotic System

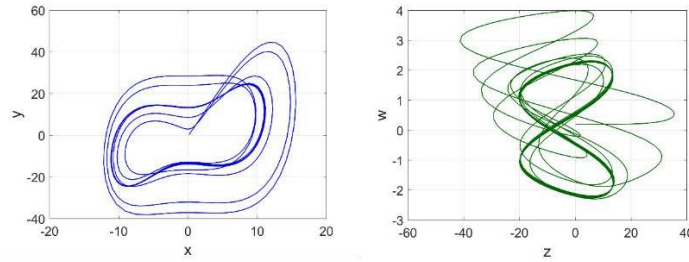
In this section, we describe a bifurcation analysis for the new chaotic system (1) introduced in Section 2. Bifurcation analysis is an important topic for studying chaotic systems [61-66].

We fix  $b = 50$  and vary  $a$  in the region of  $[2, 8]$

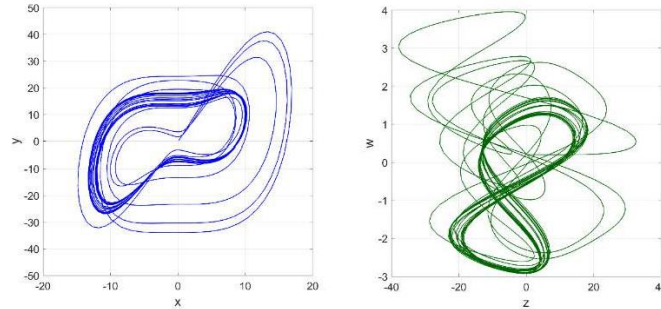
Obviously, from the bifurcation diagram shown in Figure 5, one can get that the new system (1) experiences period-doubling route to chaos. In addition, there is a big periodic-3 window in the region of  $[3.8, 4.2]$ . Some sample results are shown in Figures 6-8.



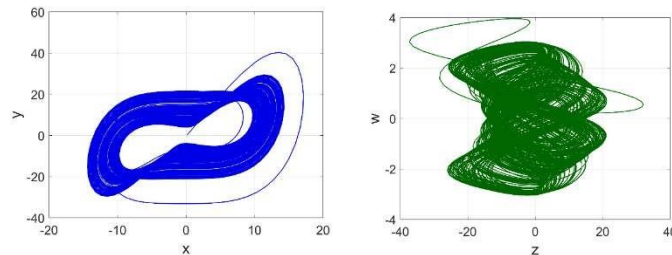
**Figure 5.** Bifurcation diagram and Lyapunov exponents of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$



**Figure 6.** Phase plot of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . When  $a = 2$ , the system (1) depicts period-1 motion.

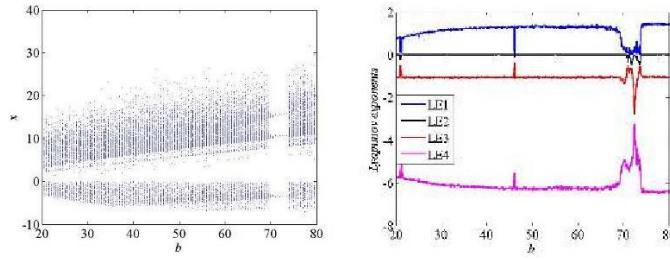


**Figure 7.** Phase plot of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . When  $a = 2.8$ , the system (1) depicts period-2 motion.



**Figure 8.** Phase plot of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . When  $a = 3$ , the system (1) depicts chaotic motion.

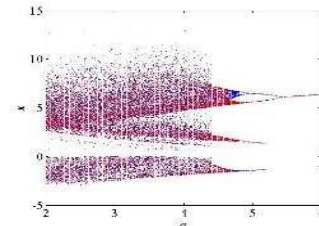
Next, we fix  $a = 5$  and vary  $b$  in the region of  $[20, 80]$ . Obviously, from the bifurcation diagram as shown in Figure 9, one can get that the system (1) is in chaotic motion except for a big periodic-3 window in the region of  $[20, 80]$ .



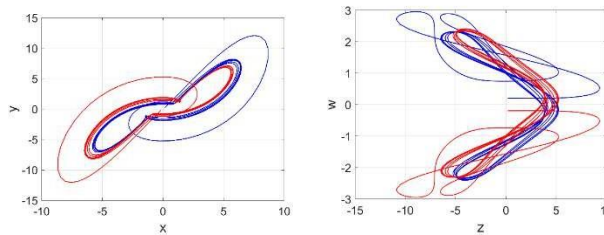
**Figure 9.** Bifurcation diagram and Lyapunov exponents of the new system (1), where we fix  $a = 5$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$

Next, we discuss on the coexisting attractors for the new system (1). We note that in Figures 10-12, the blue color refers to the phase plot of the new system (1) starting from the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and the red color refers to the phase plot of the new system (1) starting from the initial condition  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$ .

We fix  $b = 10$  and vary  $a$  in the region of  $[2, 6]$ . As can be seen from the bifurcation diagram in Figure 10 that there exists coexisting attractors in the region of  $[4.7, 5.6]$ . Some sample plots are give in Figures 11-13.



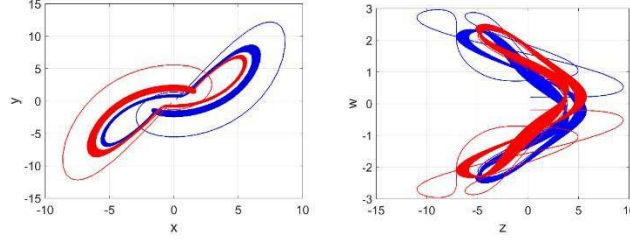
**Figure 10.** Bifurcation diagram of the new system (1), where we fix  $b = 10$  and vary  $a$  in the region of  $[2, 6]$



**Figure 11.** Phase plot of the new system (1), where we fix  $b = 10$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  for blue color plot and  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$  for red color plot. When  $a = 5$ , the system (1) has coexisting period-1 attractors.



**Figure 12.** Phase plot of the new system (1), where we fix  $b = 10$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  for blue color plot and  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$  for red color plot. When  $a = 4.9$ , the system (1) has coexisting period-2 attractors.



**Figure 13.** Phase plot of the new system (1), where we fix  $b=10$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  for blue color plot and  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$  for red color plot. When  $a = 4.75$ , the system (1) has coexisting chaotic attractors.

Next we discuss on *offset boosting control* for the new chaotic system (1).

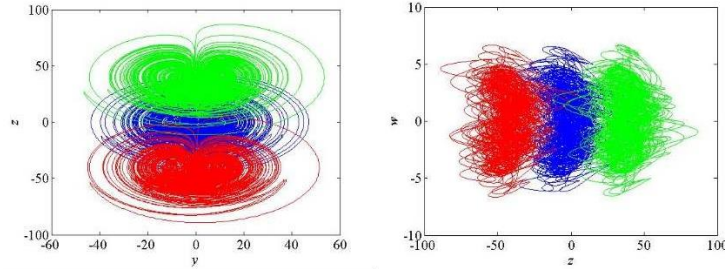
We fix  $a = 5$ ,  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ .

Clearly, the state variable  $z$  appears only once in the second equation of the new four-dimensional chaotic system (1). Thus, we can control the state variable  $z$  conveniently. The state variable  $z$  is offset-boosted by replacing  $z$  with  $z + k$ , in which  $k$  is a constant. The system (1) can be rewritten as

$$\begin{cases} \dot{x} = a(y-x) - w \\ \dot{y} = x(z+k) \\ \dot{z} = b - xy \\ \dot{w} = x - w \end{cases} \quad (8)$$

Consequently, the chaotic signal  $z$  can be transformed from a bipolar signal to a unipolar signal when varying the control parameter  $k$ .

Figure 14 shows the phase plots of the modified system (8) for  $k = 0$  (blue color),  $k = 40$  (red color) and  $k = -40$  (green color).



**Figure 14.** Phase plot of the modified system (8), where we fix  $a=5$ ,  $b=50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . In this figure, the phase plot is shown for  $k = 0$  (blue color),  $k = 40$  (red color) and  $k = -40$  (green color).

#### 4. Circuit Implementation of the New Chaotic System

The electronic circuit modelling the new four-dimensional chaotic system with only two quadratic nonlinearities (1) is realized by using off-the-shelf components such as resistors, capacitors, operational amplifiers and multipliers. We have the circuit as shown in Fig. 15 where each state variable of system (1), i.e.  $x$ ,  $y$ ,  $z$ ,  $w$  is implemented as the voltage across the corresponding capacitors  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$ , respectively.

The four state variables ( $x$ ,  $y$ ,  $z$ ,  $w$ ) of the new four-dimensional chaotic system (1) have been rescaled as  $X = \frac{1}{2}x, Y = \frac{1}{4}y, Z = \frac{1}{4}z, W = \frac{1}{2}w$ . Therefore, the the new four-dimensional chaotic system (1) is transformed into the following equivalent system:

$$\left. \begin{aligned} \dot{X} &= a(2Y - X) - W \\ \dot{Y} &= 2XZ \\ \dot{Z} &= \frac{b}{4} - 2XY \\ \dot{W} &= X - W \end{aligned} \right\} \quad (9)$$

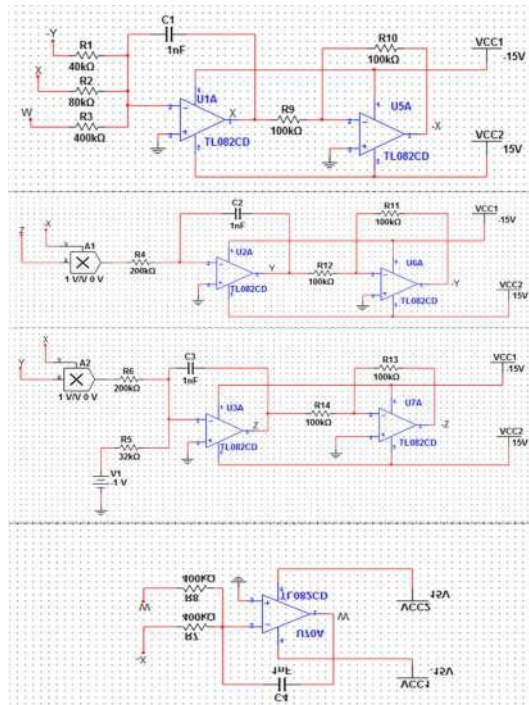
By applying Kirchoff's laws to this circuit, its dynamics are presented by the following circuit equations:

$$\left. \begin{aligned} \dot{X} &= \frac{1}{C_1 R_1} Y - \frac{1}{C_1 R_2} X - \frac{1}{C_1 R_3} W \\ \dot{Y} &= \frac{1}{C_2 R_4} XZ \\ \dot{Z} &= \frac{1}{C_3 R_5} V_1 - \frac{1}{C_3 R_6} XY \\ \dot{W} &= \frac{1}{C_4 R_7} - \frac{1}{C_4 R_8} \end{aligned} \right\} \quad (10)$$

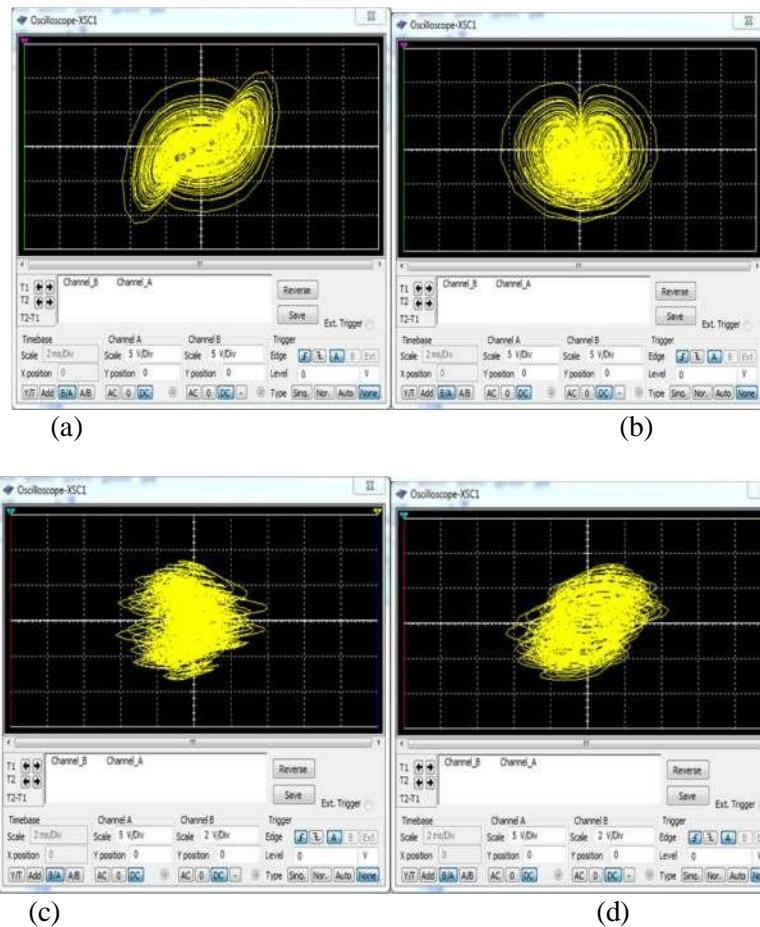
Where  $X$ ,  $Y$ ,  $Z$ ,  $W$  are the voltages across the capacitors  $C_1$ ,  $C_2$ ,  $C_3$  and  $C_4$ , respectively. The values of the circuit elements were chosen as follows:  $R_1 = 40 \text{ k}\Omega$ ,  $R_2 = 80 \text{ k}\Omega$ ,  $R_3 = R_7 = R_8 = 400 \text{ k}\Omega$ ,  $R_4 = R_6 = 200 \text{ k}\Omega$ ,  $R_5 = 32 \text{ k}\Omega$ ,  $V_1 = -1 \text{ V}_{DC}$ ,  $R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = 100 \text{ k}\Omega$ ,  $C_1 = C_2 = C_3 = C_4 = 1 \text{ nF}$ . MultiSIM simulation of the circuit are represented in Fig 16. As expected, obtained MultiSIM results confirm the theoretical model system (1).

## 5. Conclusions

A new 4-D chaotic system with only two quadratic nonlinearities was reported in this paper. It is worth noting that the new chaotic system exhibits a self-excited, two-wing strange attractor. The dynamical properties of the new chaotic system were analyzed in terms of phase portraits, equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, coexisting attractors, offset-boost control, etc. We gave a detailed analysis of the new chaotic system dynamics with bifurcation diagram and Lyapunov exponents. As an engineering application, an electronic circuit realization of the new chaotic system was designed to show the feasibility of the theoretical model.



**Figure 15.** Circuit design for the proposed new four-dimensional chaotic system (1)



**Figure 16.** MultiSIM chaotic attractors of the new four-dimensional two-wing chaotic system (1) (a) X - Y plane, (b) Y - Z plane, (c) Z - W plane and (d) X - W plane.

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# A New 4-D Chaotic System with Self-Excited Two-Wing Attractor, its Dynamical Analysis and Circuit Realization

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**Abstract.** A new four-dimensional chaotic system with only two quadratic nonlinearities is proposed in this paper. It is interesting that the new chaotic system exhibits a two-wing strange attractor. The dynamical properties of the new chaotic system are described in terms of phase portraits, equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. The new chaotic system has two saddle-foci, unstable equilibrium points. Thus, the new chaotic system exhibits self-excited attractor. Also, a detailed analysis of the new chaotic system dynamics has been carried out with bifurcation diagram and Lyapunov exponents. As an engineering application, an electronic circuit realization of the new chaotic system is designed via MultiSIM to confirm the feasibility of the theoretical 4-D chaotic model.

## 1. Introduction

In the last few decades, much attention has been devoted to the modelling and analysis of many types of chaotic systems [1-2]. Chaotic systems are very useful in many applications in science and engineering such as weather systems [3-4], ecology [5], neurons [6-7], biology [8-10], cellular neural networks [11-12], chemical reactors [13-14], oscillators [15-20], robotics [21-24], encryption [25-30], finance systems [31-32], circuits [33-45], secure communication [46-50], etc.

In the chaos literature, there is great interest about finding chaotic systems with multi-wing attractors. In the chaos literature, there are chaotic systems with two-wing attractors [51-55], three-wing attractors [56-57] and four-wing attractors [58-60].

In this research paper, we report the finding of a new 4-D chaotic system exhibiting a two-wing strange attractor. Our chaotic system has a simple structure with only two quadratic nonlinearities. We show

that the new chaotic system has two saddle-foci, unstable equilibrium points. Thus, the new chaotic system exhibits self-excited chaotic attractor [2].

We describe the phase plots of the chaotic system and conduct a rigorous bifurcation analysis of the system. Bifurcation analysis is very useful to understand the special properties of chaotic and hyperchaotic systems [61-66]. As an engineering application, we build an electronic circuit of the new chaotic system and show that the circuit experimental results of the new chaotic system show good agreement with the numerical MATLAB simulations of the system. Circuit simulation is an important work associated with a chaotic system as this can be used for applications of chaotic systems [67-70].

Section 2 describes the new chaotic system with two-wing attractor, its phase plots and Lyapunov exponents. We show that the new chaotic system has two saddle-foci equilibrium points which are unstable. Hence, we shall deduce that the new chaotic system exhibits self-excited, two-wing, chaotic attractor. Section 3 describes the dynamic analysis of the new chaotic system using bifurcation diagrams and Lyapunov exponents. Also, an electronic circuit realization of the new chaotic system is presented in detail in Section 4. Finally, Section 5 summarizes the main results derived in this work.

## 2. A new 4-D chaotic system

In this work, we report a new 4-D chaotic system given by the dynamics

$$\begin{cases} \dot{x} = a(y-x) - w \\ \dot{y} = xz \\ \dot{z} = b - xy \\ \dot{w} = x - w \end{cases} \quad (1)$$

where  $x, y, z, w$  are state variables and  $a, b$  are positive constants.  $Y=1/4, x=1/2, w=1/2, z=1/4$

We note that the system (1) has a simple structure having only two quadratic nonlinearities.

In this paper, we show that the 4-D system (1) has a *two-wing chaotic attractor* when the system parameters take the following values

$$a = 5, \quad b = 50 \quad (2)$$

For numerical simulations and calculating the Lyapunov exponents, we take the initial values of the 4-D system (1) as  $X(0) = (0.2, 0.2, 0.2, 0.2)$ . Using Wolf's algorithm [71], the Lyapunov exponents of the 4-D system (1) are found as

$$LE_1 = 1.3182, \quad LE_2 = 0, \quad LE_3 = -1.0459, \quad LE_4 = -6.2723 \quad (3)$$

Also, the Kaplan-Yorke dimension of the 4-D chaotic system (1) is determined as follows:

$$D_{KY} = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} = 3.0434 \quad (4)$$

Thus, the 4-D chaotic system exhibits complex chaotic behavior.

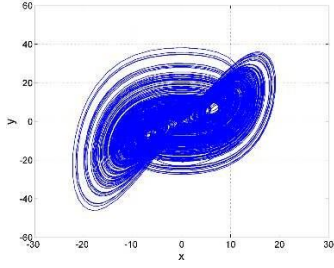
Also, the system (1) is invariant under the coordinates transformation

$$(x, y, z, w) \rightarrow (-x, -y, z, -w) \quad (5)$$

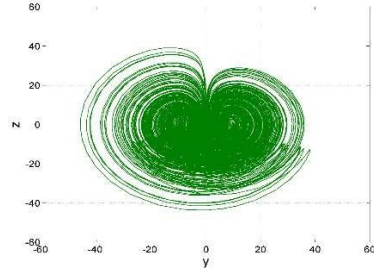
Hence, the chaotic system (1) has rotation symmetry about the  $z$ -axis. Every non-trivial trajectory of the system (1) has a twin trajectory because of this rotation symmetry property about the  $z$ -axis.

Figures 1-4 show the 2-D projections of the new chaotic system (1) in  $(x, y)$ ,  $(y, z)$ ,  $(z, w)$  and  $(x, w)$  coordinate planes, respectively.

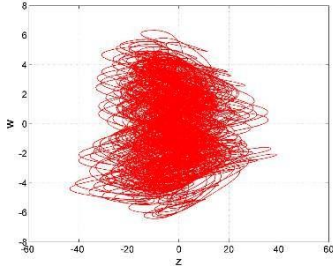
From the phase portraits, we note that the chaotic system (1) exhibits a two-wing strange attractor.



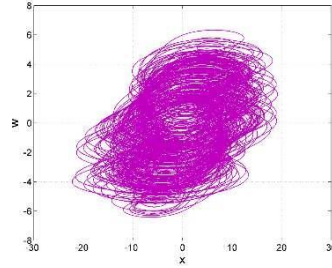
**Figure 1.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, y)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$



**Figure 2.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, y)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$



**Figure 3.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, y)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$



**Figure 4.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, y)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$

For the rest of this section, we take the values of the parameters as in the chaotic case (2), *i.e.*  $a = 5$  and  $b = 50$ .

The equilibrium points of the new chaotic system (1) are obtained by solving the system of equations

$$a(y - x) - w = 0 \quad (6a)$$

$$xz = 0 \quad (6b)$$

$$b - xy = 0 \quad (6c)$$

$$x - w = 0 \quad (6d)$$

Solving the system (6), we get two equilibrium points of the chaotic system (1) given as follows:

$$E_1 = \begin{bmatrix} 6.4550 \\ 7.7460 \\ 0 \\ 6.4550 \end{bmatrix}, E_2 = \begin{bmatrix} -6.4550 \\ -7.7460 \\ 0 \\ -6.4550 \end{bmatrix} \quad (7)$$

It is easy to verify that the equilibrium points  $E_1, E_2$  are saddle-foci, unstable, equilibrium points.

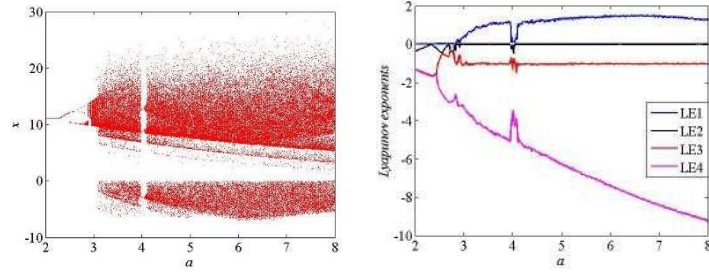
This shows that the new chaotic system (1) exhibits a self-excited, two-wing, chaotic attractor.

### 3. Bifurcation Analysis for the New Chaotic System

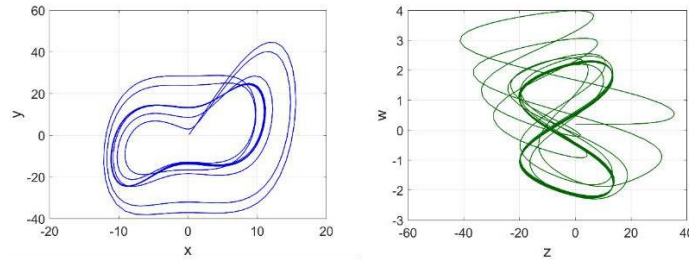
In this section, we describe a bifurcation analysis for the new chaotic system (1) introduced in Section 2. Bifurcation analysis is an important topic for studying chaotic systems [61-66].

We fix  $b = 50$  and vary  $a$  in the region of  $[2, 8]$

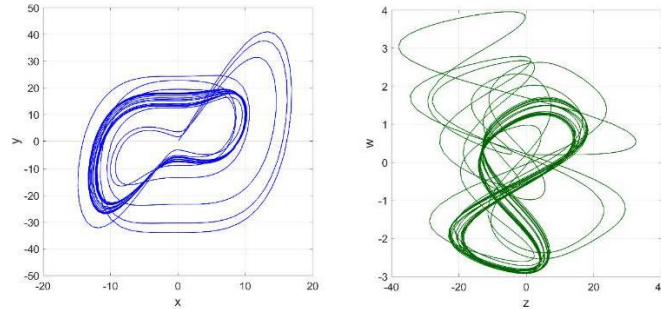
Obviously, from the bifurcation diagram shown in Figure 5, one can get that the new system (1) experiences period-doubling route to chaos. In addition, there is a big periodic-3 window in the region of  $[3.8, 4.2]$ . Some sample results are shown in Figures 6-8.



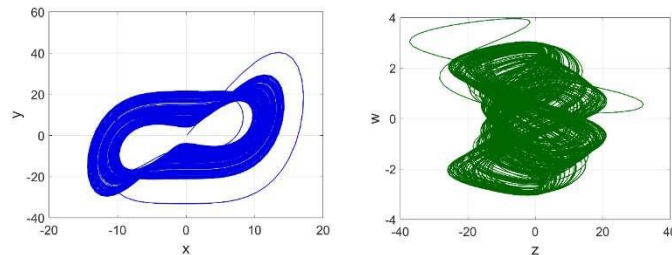
**Figure 5.** Bifurcation diagram and Lyapunov exponents of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$



**Figure 6.** Phase plot of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . When  $a = 2$ , the system (1) depicts period-1 motion.

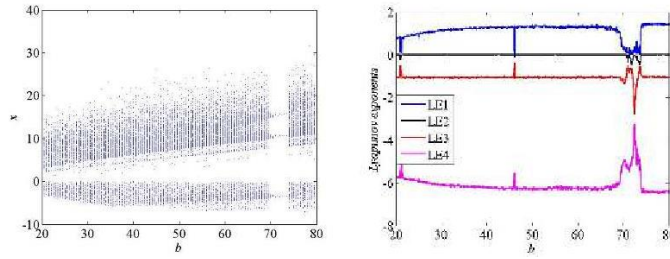


**Figure 7.** Phase plot of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . When  $a = 2.8$ , the system (1) depicts period-2 motion.



**Figure 8.** Phase plot of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . When  $a = 3$ , the system (1) depicts chaotic motion.

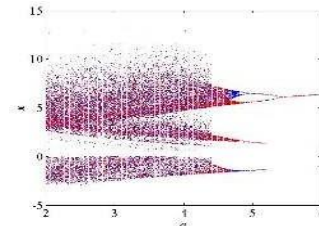
Next, we fix  $a = 5$  and vary  $b$  in the region of  $[20, 80]$ . Obviously, from the bifurcation diagram as shown in Figure 9, one can get that the system (1) is in chaotic motion except for a big periodic-3 window in the region of  $[20, 80]$ .



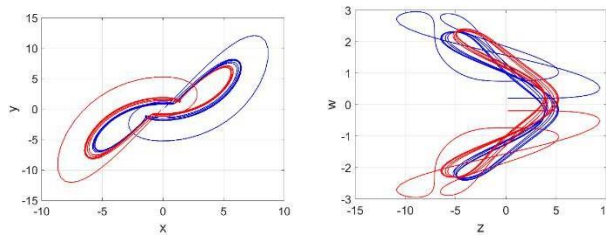
**Figure 9.** Bifurcation diagram and Lyapunov exponents of the new system (1), where we fix  $a = 5$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$

Next, we discuss on the coexisting attractors for the new system (1). We note that in Figures 10-12, the blue color refers to the phase plot of the new system (1) starting from the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and the red color refers to the phase plot of the new system (1) starting from the initial condition  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$ .

We fix  $b = 10$  and vary  $a$  in the region of  $[2, 6]$ . As can be seen from the bifurcation diagram in Figure 10 that there exists coexisting attractors in the region of  $[4.7, 5.6]$ . Some sample plots are give in Figures 11-13.



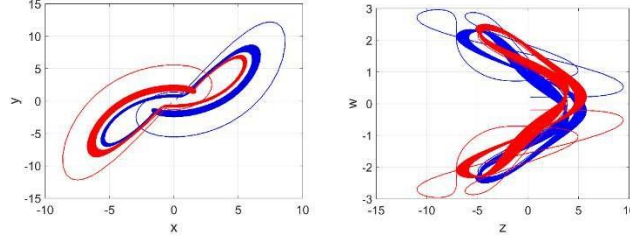
**Figure 10.** Bifurcation diagram of the new system (1), where we fix  $b = 10$  and vary  $a$  in the region of  $[2, 6]$



**Figure 11.** Phase plot of the new system (1), where we fix  $b = 10$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  for blue color plot and  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$  for red color plot. When  $a = 5$ , the system (1) has coexisting period-1 attractors.



**Figure 12.** Phase plot of the new system (1), where we fix  $b = 10$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  for blue color plot and  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$  for red color plot. When  $a = 4.9$ , the system (1) has coexisting period-2 attractors.



**Figure 13.** Phase plot of the new system (1), where we fix  $b=10$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  for blue color plot and  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$  for red color plot. When  $a = 4.75$ , the system (1) has coexisting chaotic attractors.

Next we discuss on *offset boosting control* for the new chaotic system (1).

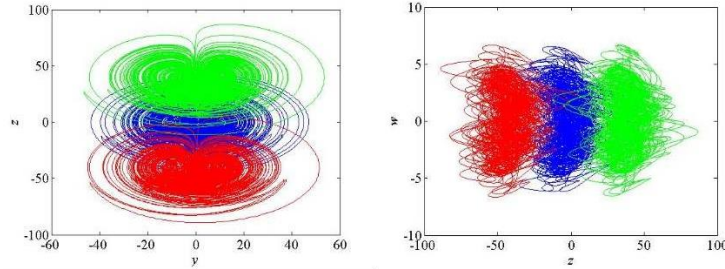
We fix  $a = 5$ ,  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ .

Clearly, the state variable  $z$  appears only once in the second equation of the new four-dimensional chaotic system (1). Thus, we can control the state variable  $z$  conveniently. The state variable  $z$  is offset-boosted by replacing  $z$  with  $z + k$ , in which  $k$  is a constant. The system (1) can be rewritten as

$$\begin{cases} \dot{x} = a(y-x) - w \\ \dot{y} = x(z+k) \\ \dot{z} = b - xy \\ \dot{w} = x - w \end{cases} \quad (8)$$

Consequently, the chaotic signal  $z$  can be transformed from a bipolar signal to a unipolar signal when varying the control parameter  $k$ .

Figure 14 shows the phase plots of the modified system (8) for  $k = 0$  (blue color),  $k = 40$  (red color) and  $k = -40$  (green color).



**Figure 14.** Phase plot of the modified system (8), where we fix  $a=5$ ,  $b=50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . In this figure, the phase plot is shown for  $k = 0$  (blue color),  $k = 40$  (red color) and  $k = -40$  (green color).

#### 4. Circuit Implementation of the New Chaotic System

The electronic circuit modelling the new four-dimensional chaotic system with only two quadratic nonlinearities (1) is realized by using off-the-shelf components such as resistors, capacitors, operational amplifiers and multipliers. We have the circuit as shown in Fig. 15 where each state variable of system (1), i.e.  $x$ ,  $y$ ,  $z$ ,  $w$  is implemented as the voltage across the corresponding capacitors  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$ , respectively.

The four state variables ( $x$ ,  $y$ ,  $z$ ,  $w$ ) of the new four-dimensional chaotic system (1) have been rescaled as  $X = \frac{1}{2}x, Y = \frac{1}{4}y, Z = \frac{1}{4}z, W = \frac{1}{2}w$ . Therefore, the the new four-dimensional chaotic system (1) is transformed into the following equivalent system:



$$\left. \begin{aligned} \dot{X} &= a(2Y - X) - W \\ \dot{Y} &= 2XZ \\ \dot{Z} &= \frac{b}{4} - 2XY \\ \dot{W} &= X - W \end{aligned} \right\} \quad (9)$$

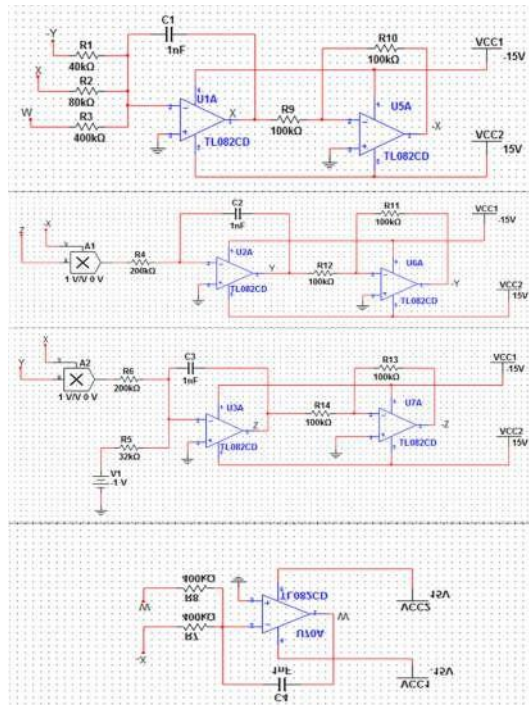
By applying Kirchoff's laws to this circuit, its dynamics are presented by the following circuital equations:

$$\left. \begin{aligned} \dot{X} &= \frac{1}{C_1 R_1} Y - \frac{1}{C_1 R_2} X - \frac{1}{C_1 R_3} W \\ \dot{Y} &= \frac{1}{C_2 R_4} XZ \\ \dot{Z} &= \frac{1}{C_3 R_5} V_1 - \frac{1}{C_3 R_6} XY \\ \dot{W} &= \frac{1}{C_4 R_7} - \frac{1}{C_4 R_8} \end{aligned} \right\} \quad (10)$$

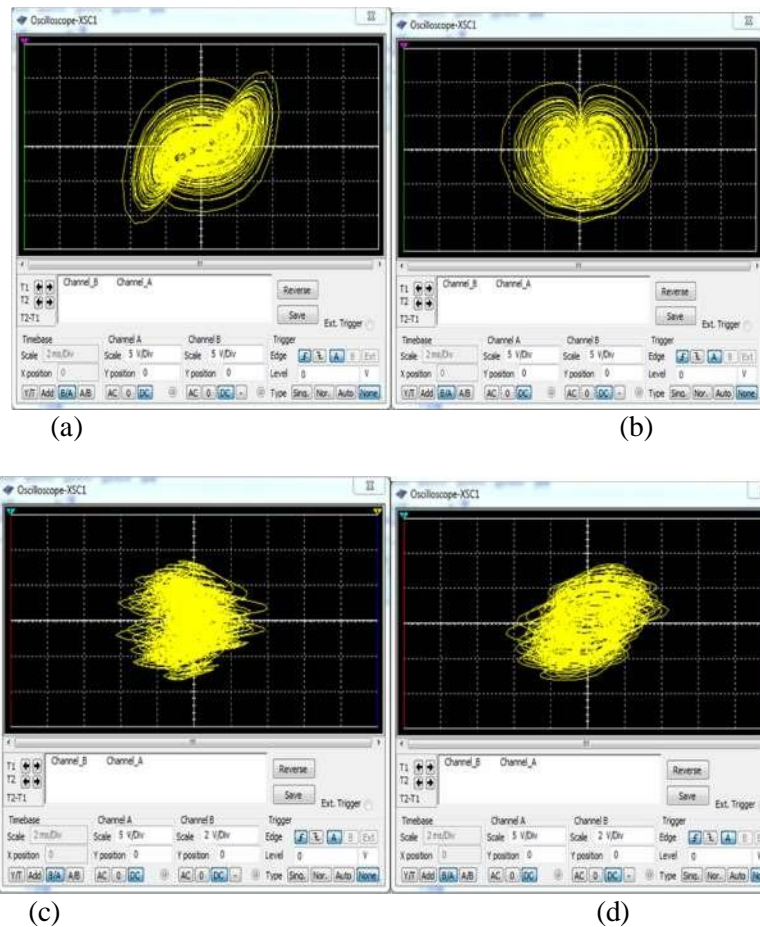
Where  $X$ ,  $Y$ ,  $Z$ ,  $W$  are the voltages across the capacitors  $C_1$ ,  $C_2$ ,  $C_3$  and  $C_4$ , respectively. The values of the circuit elements were chosen as follows:  $R_1 = 40 \text{ k}\Omega$ ,  $R_2 = 80 \text{ k}\Omega$ ,  $R_3 = R_7 = R_8 = 400 \text{ k}\Omega$ ,  $R_4 = R_6 = 200 \text{ k}\Omega$ ,  $R_5 = 32 \text{ k}\Omega$ ,  $V_1 = -1 \text{ V}_{DC}$ ,  $R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = 100 \text{ k}\Omega$ ,  $C_1 = C_2 = C_3 = C_4 = 1 \text{ nF}$ . MultiSIM simulation of the circuit are represented in Fig 16. As expected, obtained MultiSIM results confirm the theoretical model system (1).

## 5. Conclusions

A new 4-D chaotic system with only two quadratic nonlinearities was reported in this paper. It is worth noting that the new chaotic system exhibits a self-excited, two-wing strange attractor. The dynamical properties of the new chaotic system were analyzed in terms of phase portraits, equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, coexisting attractors, offset-boost control, etc. We gave a detailed analysis of the new chaotic system dynamics with bifurcation diagram and Lyapunov exponents. As an engineering application, an electronic circuit realization of the new chaotic system was designed to show the feasibility of the theoretical model.



**Figure 15.** Circuit design for the proposed new four-dimensional chaotic system (1)



**Figure 16.** MultiSIM chaotic attractors of the new four-dimensional two-wing chaotic system (1) (a) X - Y plane, (b) Y - Z plane, (c) Z - W plane and (d) X - W plane.

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# A New 4-D Chaotic System with Self-Excited Two-Wing Attractor, its Dynamical Analysis and Circuit Realization

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**Abstract.** A new four-dimensional chaotic system with only two quadratic nonlinearities is proposed in this paper. It is interesting that the new chaotic system exhibits a two-wing strange attractor. The dynamical properties of the new chaotic system are described in terms of phase portraits, equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. The new chaotic system has two saddle-foci, unstable equilibrium points. Thus, the new chaotic system exhibits self-excited attractor. Also, a detailed analysis of the new chaotic system dynamics has been carried out with bifurcation diagram and Lyapunov exponents. As an engineering application, an electronic circuit realization of the new chaotic system is designed via MultiSIM to confirm the feasibility of the theoretical 4-D chaotic model.

## 1. Introduction

In the last few decades, much attention has been devoted to the modelling and analysis of many types of chaotic systems [1-2]. Chaotic systems are very useful in many applications in science and engineering such as weather systems [3-4], ecology [5], neurons [6-7], biology [8-10], cellular neural networks [11-12], chemical reactors [13-14], oscillators [15-20], robotics [21-24], encryption [25-30], finance systems [31-32], circuits [33-45], secure communication [46-50], etc.

In the chaos literature, there is great interest about finding chaotic systems with multi-wing attractors. In the chaos literature, there are chaotic systems with two-wing attractors [51-55], three-wing attractors [56-57] and four-wing attractors [58-60].

In this research paper, we report the finding of a new 4-D chaotic system exhibiting a two-wing strange attractor. Our chaotic system has a simple structure with only two quadratic nonlinearities. We show



that the new chaotic system has two saddle-foci, unstable equilibrium points. Thus, the new chaotic system exhibits self-excited chaotic attractor [2].

We describe the phase plots of the chaotic system and conduct a rigorous bifurcation analysis of the system. Bifurcation analysis is very useful to understand the special properties of chaotic and hyperchaotic systems [61-66]. As an engineering application, we build an electronic circuit of the new chaotic system and show that the circuit experimental results of the new chaotic system show good agreement with the numerical MATLAB simulations of the system. Circuit simulation is an important work associated with a chaotic system as this can be used for applications of chaotic systems [67-70].

Section 2 describes the new chaotic system with two-wing attractor, its phase plots and Lyapunov exponents. We show that the new chaotic system has two saddle-foci equilibrium points which are unstable. Hence, we shall deduce that the new chaotic system exhibits self-excited, two-wing, chaotic attractor. Section 3 describes the dynamic analysis of the new chaotic system using bifurcation diagrams and Lyapunov exponents. Also, an electronic circuit realization of the new chaotic system is presented in detail in Section 4. Finally, Section 5 summarizes the main results derived in this work.

## 2. A new 4-D chaotic system

In this work, we report a new 4-D chaotic system given by the dynamics

$$\begin{cases} \dot{x} = a(y-x) - w \\ \dot{y} = xz \\ \dot{z} = b - xy \\ \dot{w} = x - w \end{cases} \quad (1)$$

where  $x, y, z, w$  are state variables and  $a, b$  are positive constants.  $Y=1/4, x=1/2, w=1/2, z=1/4$

We note that the system (1) has a simple structure having only two quadratic nonlinearities.

In this paper, we show that the 4-D system (1) has a *two-wing chaotic attractor* when the system parameters take the following values

$$a = 5, \quad b = 50 \quad (2)$$

For numerical simulations and calculating the Lyapunov exponents, we take the initial values of the 4-D system (1) as  $X(0) = (0.2, 0.2, 0.2, 0.2)$ . Using Wolf's algorithm [71], the Lyapunov exponents of the 4-D system (1) are found as

$$LE_1 = 1.3182, \quad LE_2 = 0, \quad LE_3 = -1.0459, \quad LE_4 = -6.2723 \quad (3)$$

Also, the Kaplan-Yorke dimension of the 4-D chaotic system (1) is determined as follows:

$$D_{KY} = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} = 3.0434 \quad (4)$$

Thus, the 4-D chaotic system exhibits complex chaotic behavior.

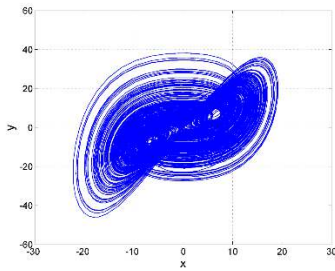
Also, the system (1) is invariant under the coordinates transformation

$$(x, y, z, w) \mapsto (-x, -y, z, -w) \quad (5)$$

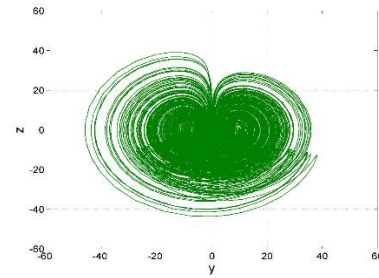
Hence, the chaotic system (1) has rotation symmetry about the  $z$ -axis. Every non-trivial trajectory of the system (1) has a twin trajectory because of this rotation symmetry property about the  $z$ -axis.

Figures 1-4 show the 2-D projections of the new chaotic system (1) in  $(x, y)$ ,  $(y, z)$ ,  $(z, w)$  and  $(x, w)$  coordinate planes, respectively.

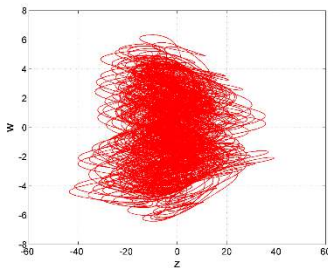
From the phase portraits, we note that the chaotic system (1) exhibits a two-wing strange attractor.



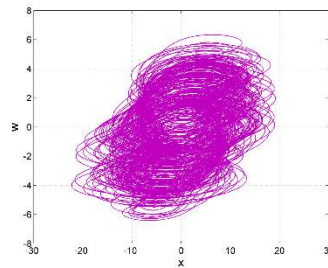
**Figure 1.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, y)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$



**Figure 2.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, y)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$



**Figure 3.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, y)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$



**Figure 4.** 2-D phase plot of the 4-D chaotic system (1) in the  $(x, y)$  plane for  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and  $(a, b) = (5, 50)$

For the rest of this section, we take the values of the parameters as in the chaotic case (2), *i.e.*  $a = 5$  and  $b = 50$ .

The equilibrium points of the new chaotic system (1) are obtained by solving the system of equations

$$a(y - x) - w = 0 \tag{6a}$$

$$xz = 0 \tag{6b}$$

$$b - xy = 0 \tag{6c}$$

$$x - w = 0 \tag{6d}$$

Solving the system (6), we get two equilibrium points of the chaotic system (1) given as follows:

$$E_1 = \begin{bmatrix} 6.4550 \\ 7.7460 \\ 0 \\ 6.4550 \end{bmatrix}, E_2 = \begin{bmatrix} -6.4550 \\ -7.7460 \\ 0 \\ -6.4550 \end{bmatrix} \tag{7}$$

It is easy to verify that the equilibrium points  $E_1, E_2$  are saddle-foci, unstable, equilibrium points.

This shows that the new chaotic system (1) exhibits a self-excited, two-wing, chaotic attractor.

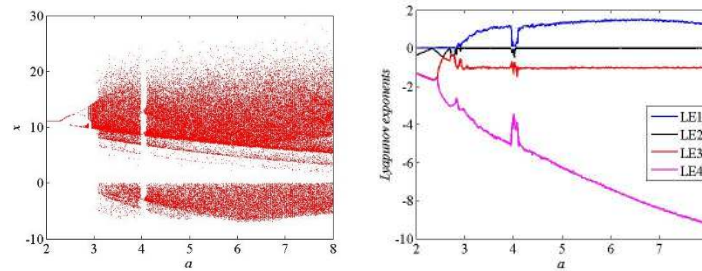
### 3. Bifurcation Analysis for the New Chaotic System

In this section, we describe a bifurcation analysis for the new chaotic system (1) introduced in Section 2. Bifurcation analysis is an important topic for studying chaotic systems [61-66].

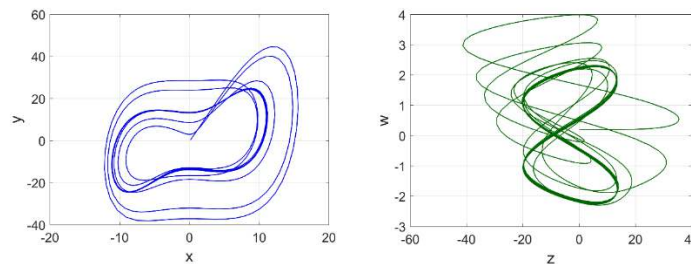
We fix  $b = 50$  and vary  $a$  in the region of  $[2, 8]$

Obviously, from the bifurcation diagram shown in Figure 5, one can get that the new system (1) experiences period-doubling route to chaos. In addition, there is a big periodic-3 window in the region of  $[3.8, 4.2]$ . Some sample results are shown in Figures 6-8.

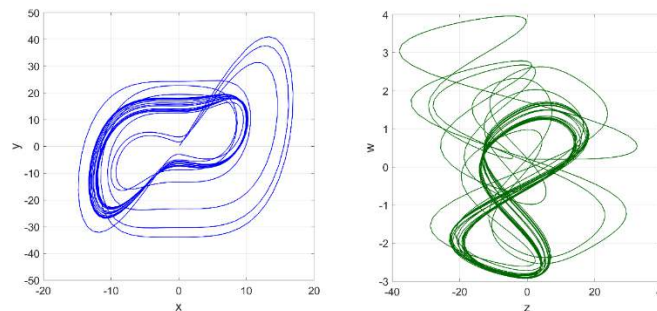




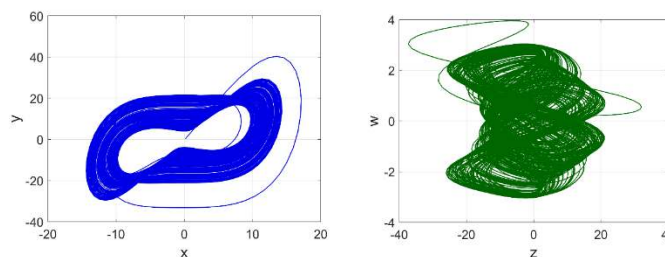
**Figure 5.** Bifurcation diagram and Lyapunov exponents of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$



**Figure 6.** Phase plot of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . When  $a = 2$ , the system (1) depicts period-1 motion.

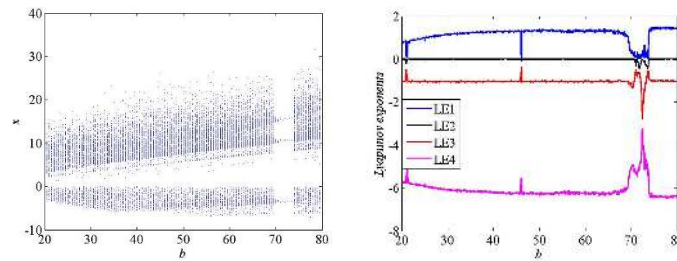


**Figure 7.** Phase plot of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . When  $a = 2.8$ , the system (1) depicts period-2 motion.



**Figure 8.** Phase plot of the new system (1), where we fix  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . When  $a = 3$ , the system (1) depicts chaotic motion.

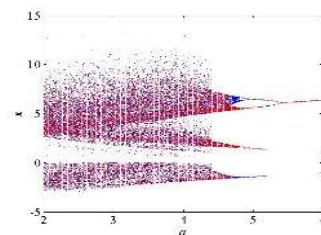
Next, we fix  $a = 5$  and vary  $b$  in the region of  $[20, 80]$ . Obviously, from the bifurcation diagram as shown in Figure 9, one can get that the system (1) is in chaotic motion except for a big periodic-3 window in the region of  $[20, 80]$ .



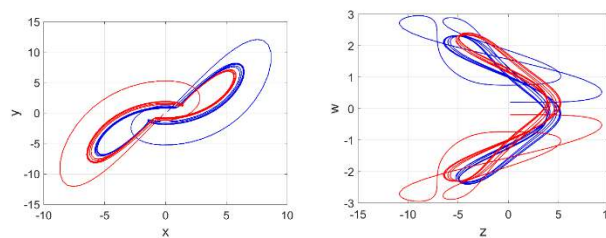
**Figure 9.** Bifurcation diagram and Lyapunov exponents of the new system (1), where we fix  $a = 5$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$

Next, we discuss on the coexisting attractors for the new system (1). We note that in Figures 10-12, the blue color refers to the phase plot of the new system (1) starting from the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  and the red color refers to the phase plot of the new system (1) starting from the initial condition  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$ .

We fix  $b = 10$  and vary  $a$  in the region of  $[2, 6]$ . As can be seen from the bifurcation diagram in Figure 10 that there exists coexisting attractors in the region of  $[4.7, 5.6]$ . Some sample plots are give in Figures 11-13.



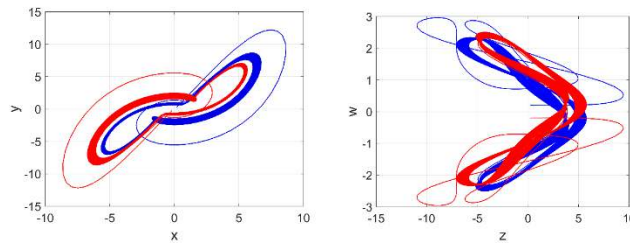
**Figure 10.** Bifurcation diagram of the new system (1), where we fix  $b = 10$  and vary  $a$  in the region of  $[2, 6]$



**Figure 11.** Phase plot of the new system (1), where we fix  $b = 10$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  for blue color plot and  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$  for red color plot. When  $a = 5$ , the system (1) has coexisting period-1 attractors.



**Figure 12.** Phase plot of the new system (1), where we fix  $b = 10$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  for blue color plot and  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$  for red color plot. When  $a = 4.9$ , the system (1) has coexisting period-2 attractors.



**Figure 13.** Phase plot of the new system (1), where we fix  $b = 10$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$  for blue color plot and  $Y_0 = (-0.2, -0.2, 0.2, -0.2)$  for red color plot. When  $a = 4.75$ , the system (1) has coexisting chaotic attractors.

Next we discuss on *offset boosting control* for the new chaotic system (1).

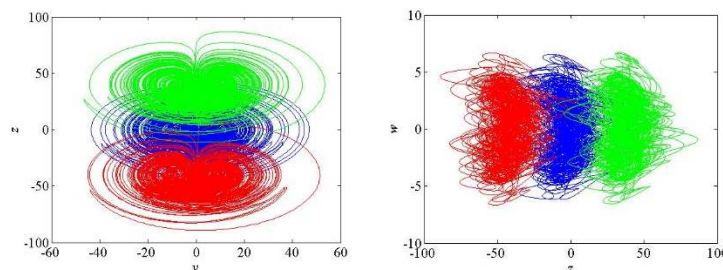
We fix  $a = 5$ ,  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ .

Clearly, the state variable  $z$  appears only once in the second equation of the new four-dimensional chaotic system (1). Thus, we can control the state variable  $z$  conveniently. The state variable  $z$  is offset-boosted by replacing  $z$  with  $z + k$ , in which  $k$  is a constant. The system (1) can be rewritten as

$$\begin{cases} \dot{x} = a(y - x) - w \\ \dot{y} = x(z + k) \\ \dot{z} = b - xy \\ \dot{w} = x - w \end{cases} \quad (8)$$

Consequently, the chaotic signal  $z$  can be transformed from a bipolar signal to a unipolar signal when varying the control parameter  $k$ .

Figure 14 shows the phase plots of the modified system (8) for  $k = 0$  (blue color),  $k = 40$  (red color) and  $k = -40$  (green color).



**Figure 14.** Phase plot of the modified system (8), where we fix  $a = 5$ ,  $b = 50$  and the initial condition  $X_0 = (0.2, 0.2, 0.2, 0.2)$ . In this figure, the phase plot is shown for  $k = 0$  (blue color),  $k = 40$  (red color) and  $k = -40$  (green color).

#### 4. Circuit Implementation of the New Chaotic System

The electronic circuit modelling the new four-dimensional chaotic system with only two quadratic nonlinearities (1) is realized by using off-the-shelf components such as resistors, capacitors, operational amplifiers and multipliers. We have the circuit as shown in Fig. 15 where each state variable of system (1), i.e.  $x$ ,  $y$ ,  $z$ ,  $w$  is implemented as the voltage across the corresponding capacitors  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$ , respectively.

The four state variables ( $x$ ,  $y$ ,  $z$ ,  $w$ ) of the new four-dimensional chaotic system (1) have been rescaled as  $X = \frac{1}{2}x, Y = \frac{1}{4}y, Z = \frac{1}{4}z, W = \frac{1}{2}w$ . Therefore, the the new four-dimensional chaotic system (1) is transformed into the following equivalent system:

$$\left. \begin{aligned} \dot{X} &= a(2Y - X) - W \\ \dot{Y} &= 2XZ \\ \dot{Z} &= \frac{b}{4} - 2XY \\ \dot{W} &= X - W \end{aligned} \right\} \quad (9)$$

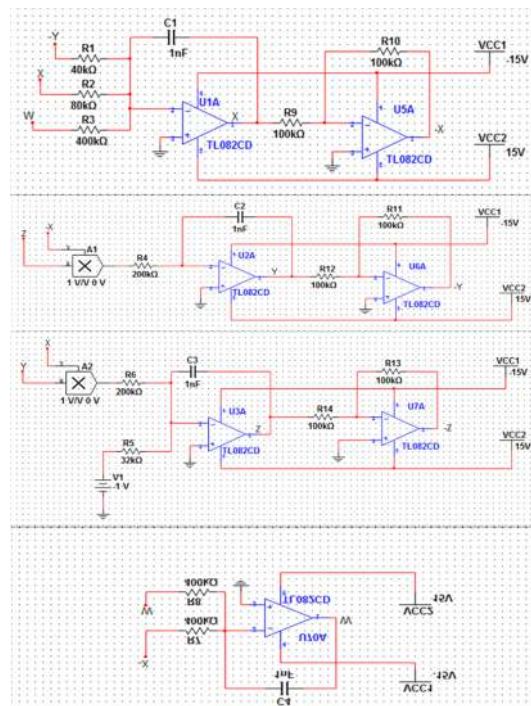
By applying Kirchoff's laws to this circuit, its dynamics are presented by the following circuital equations:

$$\left. \begin{aligned} \dot{X} &= \frac{1}{C_1 R_1} Y - \frac{1}{C_1 R_2} X - \frac{1}{C_1 R_3} W \\ \dot{Y} &= \frac{1}{C_2 R_4} XZ \\ \dot{Z} &= \frac{1}{C_3 R_6} V_1 - \frac{1}{C_3 R_5} XY \\ \dot{W} &= \frac{1}{C_4 R_7} - \frac{1}{C_4 R_8} \end{aligned} \right\} \quad (10)$$

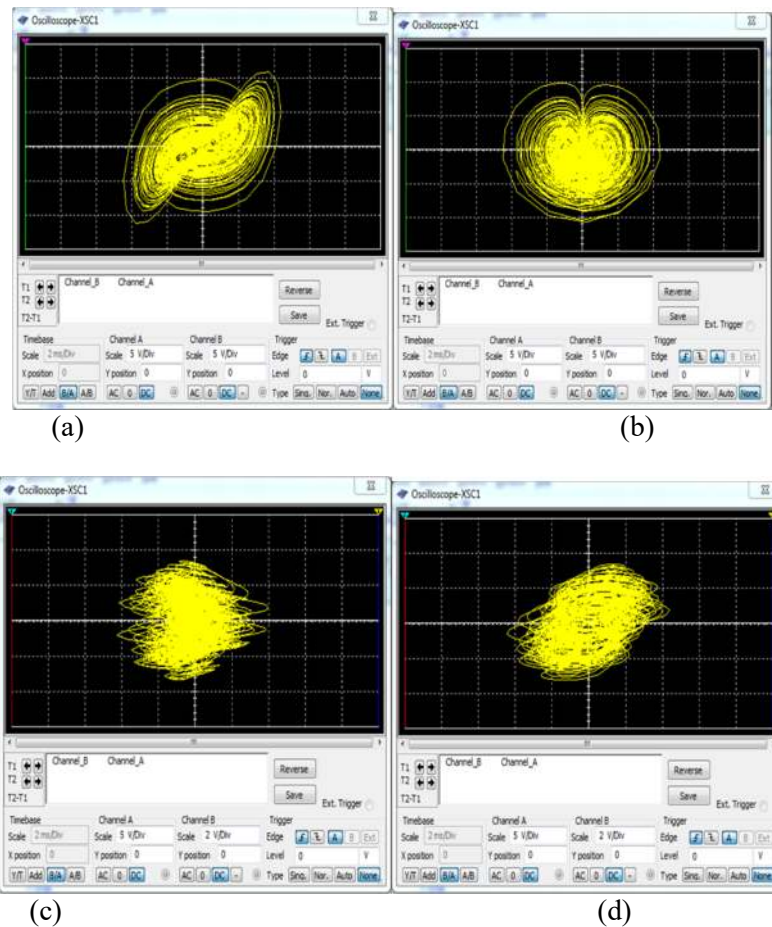
Where  $X, Y, Z, W$  are the voltages across the capacitors  $C_1, C_2, C_3$  and  $C_4$ , respectively. The values of the circuit elements were chosen as follows:  $R_1 = 40 \text{ k}\Omega, R_2 = 80 \text{ k}\Omega, R_3 = R_7 = R_8 = 400 \text{ k}\Omega, R_4 = R_6 = 200 \text{ k}\Omega, R_5 = 32 \text{ k}\Omega, V_1 = -1 \text{ V}_{DC}, R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = 100 \text{ k}\Omega, C_1 = C_2 = C_3 = C_4 = 1 \text{ nF}$ . MultiSIM simulation of the circuit are represented in Fig 16. As expected, obtained MultiSIM results confirm the theoretical model system (1).

### 5. Conclusions

A new 4-D chaotic system with only two quadratic nonlinearities was reported in this paper. It is worth noting that the new chaotic system exhibits a self-excited, two-wing strange attractor. The dynamical properties of the new chaotic system were analyzed in terms of phase portraits, equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, coexisting attractors, offset-boost control, etc. We gave a detailed analysis of the new chaotic system dynamics with bifurcation diagram and Lyapunov exponents. As an engineering application, an electronic circuit realization of the new chaotic system was designed to show the feasibility of the theoretical model.



**Figure 15.** Circuit design for the proposed new four-dimensional chaotic system (1)



**Figure 16.** MultiSIM chaotic attractors of the new four-dimensional two-wing chaotic system (1) (a)  $X - Y$  plane, (b)  $Y - Z$  plane, (c)  $Z - W$  plane and (d)  $X - W$  plane.

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